let us recognise the absurdity and illogicality of the age restriction. It is ironic that the very organisations which have done so much to instigate and support the trend towards a greater equality of educational opportunity for the mass of the population should be the ones to shut the door in the face of their fellow workers who wish to enter their ranks. Many semi-skilled men who regret their lost opportunities could do the more skilled jobs if they were given the chance to get the necessary training ; it is only the exercise of a harsh monopoly power which prevents them from doing so. If recognition as a skilled craftsman depended on passing a test of competence instead of, as now, solely on the length of training at a particular age, a semi-skilled operative might, by working at a technical institute in the evenings, get the opportunity to better himself. This would be both more just to him and better for industry which needs him.

Second we must accept the fact that there are different degrees of intellectual ability as well as manual skill. To assume that every boy who gets accepted for an apprenticeship (a process which is rarely more than casually selective) is necessarily capable of mastering the technical instruction which was planned for the most intelligent and enthusiastic pupils is as illogical as to assume that every boy who gets into a grammar school is capable of taking an honours degree at a university. We must recognise a heirarchy of talent and provide for it accordingly. This does not mean that the less intelligently endowed apprentice does not need to go to school or college during his training. On the contrary, he does. But to set him to a task which is quite beyond his power is to make him bored and resentful and distrustful of any educational effort. The County Colleges planned by the Education Act were intended to offer a wider and more flexible curriculum than the merely technical. They were designed to give young people some understanding of the society in which they live and of their own part and that of their industry, in it. If the apprentices who now attend the classes in which they are completely lost could instead be provided with something more closely tailored to their needs the day at college could be made both enjoyable and valuable.

It is possible that even for the abler boys who can profit from the course, a different arrangement of time might be more suitable. For apprentices of a higher grade—those aiming for technical jobs—a "sandwich" system has been introduced in which a period of months in the shops is followed by a period of full time study at college. It might be that a similar system for the craftsman would be more effective. Five months at work followed by one month at college would provide just the same number of study hours as the present system of day release ; but it would give the colleges a chance to plan a course of instruction designed to meet the needs of the apprentice and time to carry it into effect.

Such changes, valuable as they may be in themselves, would not, however, solve the problem of the shortage of numbers. So long as adolescents' wages are as high as at present it is unlikely that many more small or moderate sized firms will be willing to increase the number of their apprentices. Two possible ways out of this difficulty may be suggested. Groups of firms might join together to share the cost of a number of apprentices who move from one firm to another for their training. This is already being done on a small scale in London and in the Midlands. The main disadvantage is that it takes a lot of organising and it may cost as much time and money to persuade a large number of such schemes to be set up as the expense of apprenticeship to individual firms.

A more constructive proposal because it can be done on a large scale is to substitute apprenticeship to a whole industry for apprenticeship to an individual employer. This is done in America in the building industry and overcomes the unwillingness of a firm to face the risk involved in becoming responsible for apprentices for a long period of time in a kind of production that is bound to be to some extent irregular. To be successful, such a scheme would need to be run by a Joint Apprenticeship Council composed of representatives of both unions' and employers' associations, which would make itself responsible for paying the wages of the apprentices indentured to it and for supervising their training. As the funds would be provided by employers in proportion to their wages bill the cost of training the skilled workers on whom all depend would be spread more fairly over the whole industry instead of, as now, being concentrated on those firms that have been willing to undertake their responsibilities voluntarily.

Such a scheme would have an additional advantage. It is often difficult for the small firm, with the best will in the world, to give its apprentices an all-round training because its own work may be highly specialised. Apprenticeship to an industry would get over this difficulty because the trainee could be moved from firm to firm.

It is much to be hoped that the Carr Committee Report will discuss some of these possibilities. But the first essential is for industry and unions to face the realities of the present situation and be prepared to consider a radical reorganisation.

Theory of Regenerative Machine Tool Chatter*

By S. A. TOBIAS, D.Sc., Ph.D.,† and PROFESSOR W. FISHWICK, Ph.D.‡

No. I

ALTHOUGH an appreciable amount of research has been done already on machine tool chatter, little use is made of the results by production engineers and machine tool designers. This appears to be due to three reasons : (i) some of the recommendations based on theoretical considerations appear to be contrary to practical experience ; (ii) owing to the large number of parameters involved in chatter phenomena the theoretical recommendations for the avoidance of chatter are necessarily of great complexity and they have not been presented in a form suitable for direct application in the design office ; (iii) although it is known that chatter occurring in the various types of machine tools is essentially due to the same basic physical phenomena, at the present time there does not exist a general theory which would cover all the special cases. As a result chatter arising in each of the various types of machine tools must be treated as an individual problem, thus increasing the difficulty of applying the theoretical recommendations.

It has been pointed out that by increasing the stiffness between work piece and cutting tool the danger of chatter is decreased. At the same time, as every production engineer knows, chatter is often eliminated by easing some of the locks of machine tool slides, &c., and thus increasing the structural flexibility. Designers are aware that some flexible machines appear to perform better from the point of view of chatter than similar machines with appreciably greater stiffness. The answer given by the research worker is that in chatter phenomena damping also plays a highly important part and that in some cases a loss of stiffness may be accompanied by an increase of damping, resulting in an elimination of chatter. Unfortunately, this explanation is not satisfactory even from the theoretical point of view since the quantitative relationship between these two factors is at present unknown.

It is well known that, in addition to stiffness and damping, the cutting speed, the feed, the geometrical form of the tool, the work piece material, the foundation of the structure, &c., all play an important part in chatter. However, the individual effect of these parameters is insufficiently explored and so in a particular case it is difficult to say which parameter should be altered and in what way, in order to eliminate any chatter. Thus, as a rule, chatter is eliminated by time and money-consuming random experimentation. It is clear that owing to the large number of parameters which have an influence on the chatter behaviour of a machine, a theory which would consider all essential factors would be of such enormous complexity that there is little hope for its practical application in the drawing-office or in the workshop. This difficulty can be overcome only by the introduction of "group parameters" upon which a relatively simple mathematical theory can be built. It would be an additional advantage if the final results of such a theory could be presented in a graphical form, thus eliminating the necessity of time-consuming laborious calculations. Furthermore, it is desirable that the mathematical part of this theory should be directly valid for all kinds of chatter occurring on the various kinds of machine tools and that individual cases should be derived from it by a substitution of the operative values of the group parameters corresponding to the particular type of machining under consideration. Such a unified picture of the chatter phenomena would not only facilitate the application of the theoretical results, but would also permit the rapid solution of unusual cases previously not investigated.

A theory for drilling chatter which to some extent satisfies these demands has recently been published by the authors (subsequently referred to as [1]).§ In the present article this theory will be extended to include other kinds of machining processes.

* This article is an extended version of a paper read at the "Second Colloquium for Machine Tool Research and Design" (2.FoKoMa), held at the Technische Hochschule, München, 1955.

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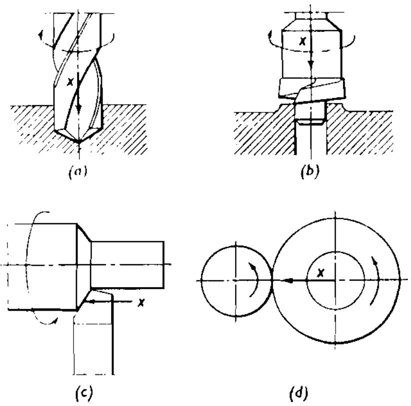
§ References are given at the end of this article.

Scope of the Theory

The present theory is mainly concerned with regenerative chatter, but other types will also be included. Regenerative chatter occurs when part of the chip removed by a cutting edge has previously been cut, either by the same cutting edge (single edge tools, such as lathe tools or grinding wheels), or by another cutting edge which is solidly connected to the first (multi-edge tools, such as drills, face millers, &c.). In this case the forces acting on the cutting edge at a certain time instant are partly determined by the form of the chip removed at an earlier time. We shall restrict the scope of the exposition by confining ourselves to those kinds of machining in which under chatter-free conditions the chip thickness removed is constant.



(*a*) drilling ; (*b*) face milling ; (*c*) turning ; (*d*) grinding. It is assumed that the chatter amplitudes fall in the direction *x*.

Fig. 1—Cutting processes embraced by the theory

Thus, we shall be concerned with drilling, certain types of face milling, turning (shaping) and grinding, as shown on Fig. 1.

In the cases shown on Fig. 1 the cutting edge has three rectilinear degrees of freedom which fall in the direction of the feed motion, the direction of the cutting velocity, and the direction which is perpendicular to the other two. Chatter confined to the plane perpendicular to the cutting velocity will be called type-A chatter, irrespective whether its direction is given by the feed motion (denoted by x in Fig. 1(*a*) to 1(*c*)) or the direction perpendicular to it. Type-B chatter is entirely confined to the direction of the cutting velocity. In the present paper we shall consider only type-A chatter and the more general theory which treats both types will be left for a future publication.

A Theory of Regenerative Chatter

Consider a machining operation in process with a nominal feed s_0 inch/rev. and v_0 = constant cutting velocity. Under these conditions the particular machine tool can be regarded as a dynamical system which is in steady motion. It will now be investigated whether this motion is stable or unstable—that is, in the terminology of the production engineer, whether the machining will be free of chatter or not. This will be done by assuming that there is a relative vibration x between work piece and cutting edge which falls in the directions shown on Fig. 1(*a*) to 1(*d*). This vibration will be expressed as

$$x = Ae^{\alpha t} \cos \omega t \quad . \quad . \quad . \quad . \quad (1)$$

where A is an arbitrary amplitude factor and α and ω are at present unknown, but will be determined in the course of the calculation. We shall investigate for which values of the system parameters $\alpha \lesseqgtr 0$. Clearly, when $\alpha < 0$ equation (1) represents a damped oscillation and as t increases $x \rightarrow 0$ and so the nominal feed establishes itself. In this case we shall say that machining takes place under stable conditions. When $\alpha > 0$ the superimposed vibration increases in amplitude as time goes on and tends to infinity. In practice, some limiting factor will stabilise the amplitude at a finite but unpleasantly large value. The case $\alpha > 0$ signifies unstable machining conditions. Finally, when $\alpha = 0$ the system is on the threshold of stability and in that case, once initiated, the vibration amplitude remains at its initial level.

It must be emphasised that throughout the following analysis no attention is paid to the question as to how energy can be fed into the system or how the amplitudes build up and stabilise themselves at their final level. In accordance with the procedure usually adopted for the investigation of the stability of dynamic systems, the view is taken that if the values of the system parameters are such that the system is unstable, the smallest disturbance (a hard spot in the material, unbalance vibrations, &c.) is sufficient to induce it to leave its steady state of motion and to burst into oscillations (to chatter). Thus the analysis is only concerned with the problem of stability—that is, whether or not the machine will chatter under certain working conditions.

The general procedure of the method employed can now be summarised as follows : the presence of the assumed vibration x entails an alteration of the cutting conditions as a result of which superimposed on the steady state cutting thrust component P_x a thrust increment dP_x is generated. Since dP_x is caused by x, it will be a function of x and so of the time t ; dP_x is acting on the elastic components of the system (work piece and cutting tool) and it may be such as to either increase x or decrease it or leave it on its original level. In the first case the calculation will yield $\alpha > 0$ and then the system is unstable, i.e. chatter will occur. In the second case $\alpha < 0$ which indicates stability and finally in the third case $\alpha = 0$, which shows that the system is on the threshold of stability.

It will be assumed that the cutting force thrust component P_x which falls in the direction x, according to Fig. 1, is a function of the instantaneous values of the following four parameters : the total chip width b inch, the total chip thickness s inch/rev., the rate of penetration (feed velocity) r inch/sec, and the cutting speed v inch/sec; v is dependent on the rotational speed of the tool or work-piece Ω rev./sec as $v = 2\pi R\Omega$, where R is the radius from the centre of rotation. Assuming that the chip width b remains unaffected by the presence of the vibration x, it can be neglected from further considerations. Under steady state machining conditions the remaining three parameters are related by $r = s_0\Omega$ and then the total chip thickness s is equal to the nominal feed s_0. In this case it is sufficient to say that the thrust P_x is a function of s_0 and Ω. However, under dynamic conditions the chip thickness s may vary independently of the feed rate r and both may be only partly determined by the nominal feed s_0, as we shall see presently in greater detail. Thus, in general, the thrust can be written $P_x = P(s, r, \Omega)$ and so the incremental thrust can be expressed as

$$dP_x = k_1 ds + k_2 dr + k_3 d\Omega \quad . \quad . \quad (2)$$

where $k_1 = (dP_x/ds)_0$ represents the increase of the thrust per unit increase of chip thickness, all other variables remaining constant. $k_2 = (dP_x/dr)_0$ and $k_3 = (dP_x/d\Omega)_0$ have an analogous meaning with respect to the feed rate and the cutting speed, respectively. All three constants are evaluated under steady state conditions.

Consider changes from one steady state to another under the following experimental conditions : (i) steady state machining tests are being carried out at constant speed ($d\Omega = 0$) and only the nominal feed s_0 (which in this case is equal to the total chip thickness) is varied. By plotting the thrust P_x as a function of the feed a graph of the kind shown on Fig. 2(*a*) is obtained. From

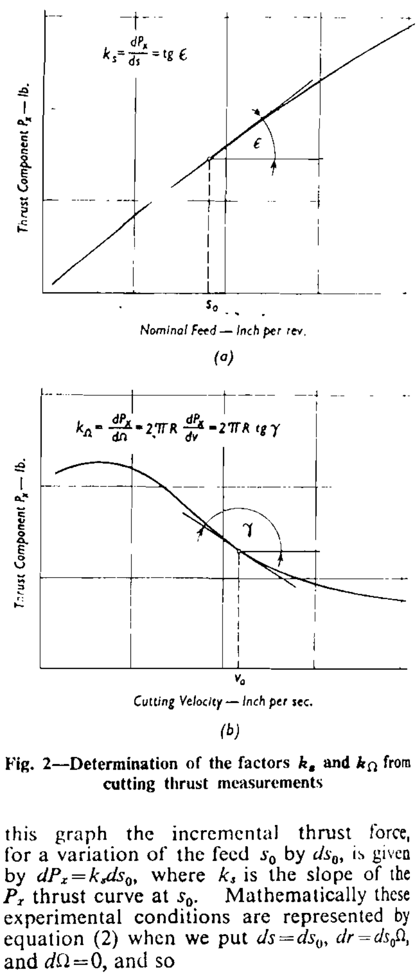


Fig. 2—Determination of the factors k_s and k_Ω from cutting thrust measurements

this graph the incremental thrust force, for a variation of the feed s_0 by ds_0, is given by $dP_x = k_s ds_0$, where k_s is the slope of the P_x thrust curve at s_0. Mathematically these experimental conditions are represented by equation (2) when we put $ds = ds_0$, $dr = ds_0\Omega$, and $d\Omega = 0$, and so

$$dP_x = k_1 ds_0 + k_2 \Omega ds_0 = k_s ds_0 \quad . \quad . \quad (3)$$

from which

$$k_s = k_1 + k_2\Omega \quad . \quad\quad (4)$$

(ii) Consider now the case when the feed is kept constant and the tool or work piece speed is varied. By plotting the P_x thrust as a function of v_0 we obtain in some cases a curve of the kind shown on Fig. 2(*b*). Mathematically, when $ds = 0$ but $d\Omega \neq 0$ and so $dr = s_0 d\Omega$ equation (2) yields

$$k_\Omega d\Omega = 0 + k_2 dr + k_3 d\Omega \quad . \quad\quad (5)$$

and so

$$k_3 = k_\Omega - k_2 s_0 = k_\Omega - (k_s - k_1)(s_0/\Omega) \quad\quad (6)$$

Thus equation (2) can be written :

$$dP_x = k_1 ds + (k_s - k_1)\frac{dr}{\Omega} + \left[k_\Omega - (k_s - k_1)\frac{s_0}{\Omega}\right]d\Omega$$
$$. \quad . \quad . \quad . \quad . \quad (7)$$

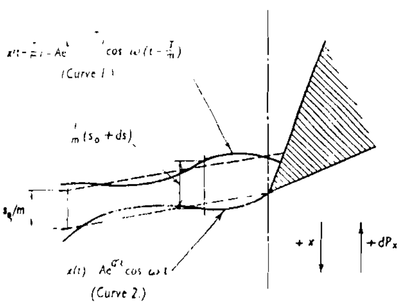
We shall write $(k_s - k_1) = K$. In equation (7) there are three constants, k_1, k_s and k_Ω, which have to be determined experimentally. The experimental evaluation of k_s and k_Ω is obvious from the derivation of the equation. However, the evaluation of k_1 is not so simple, partly because it is essentially a dynamic factor which does not appear in the work of previous investigators. This is not surprising when it is considered that their work is concerned with steady state machining conditions. In this case the nominal feed s_0 and the feed rate are related by $r = s_0\Omega$, as has already been pointed out. However, k_1 can be determined only from experiments in which the chip thickness s and the feed rate r are independently varied. The design of such experiments is by no means an easy matter and needs further discussion. An approximate evaluation for k_1 for the case of drilling has been given by the authors elsewhere.[1]

Assuming the values of k_1, k_s and k_Ω are known, dP_x is found by calculating the chip thickness variation ds, the feed rate variation dr and the rotational speed variation $d\Omega$.

Type-A Chatter

In this case the rotational speed Ω is not directly affected by the presence of x. Speed variations may occur owing to the variation of the load which may set up vibration in the spindle drive, but this possibility will be disregarded and so we put $d\Omega = 0$.

The chip thickness variations occurring in the types of machining shown on Fig. 1(*a*) to 1(*c*) will be explained on Fig. 3. This



The instantaneous chip thickness is given by $(1/m)(s_0+ds)$.

Fig. 3—Variation of the chip thickness

figure shows a section through one of the cutting edges at time t in a plane including cutting speed vector and the direction x. If the nominal feed be s_0 then under steady state conditions each cutting edge removes a chip of uniform thickness s_0/m, where m is the number of cutting edges of the tool. The form of this chip is indicated by the dotted lines in the figure. Owing to the presence of the vibration x the upper surface of the chip has been cut in the form of a wave (curve 1) when it was machined last by the cutting edge which directly preceded the edge shown in the figure. If the time of one revolution of the tool (drilling and face milling) or of the work piece (turning and grinding) is $T = 1/\Omega$ then the upper surface of the chip was last cut at time $t - T/m$. As at time t the cutting edge proceeds along curve 2. The instantaneous chip thickness removed is given by $(s_0+ds)/m$, where

$$(1/m)ds = x(t) - \mu x(t - T/m) \quad . \quad . \quad (8)$$

where μ is the factor of overlapping between the previous cut and the present cut. Clearly, in the case of drilling $m = 2$ (there being two cutting edges) and $\mu = 1\cdot0$ since the whole width of the top surface of the chip has been modulated by the previous cut. For face milling we usually have $m = 3$, 4, &c., but still $\mu = 1\cdot0$. In the case of turning $m = 1$ and $\mu \leq 1\cdot0$. For grinding we have also $m = 1$ but the value of μ depends on whether the direction of the feed motion is perpendicular to the work piece surface ($\mu = 1\cdot0$) or is parallel to it ($\mu < 1\cdot0$). It should be added that $\mu = 0$ for all types of machining under consideration during the first $1/m$-th revolution of the tool or the work piece. Thus, the theory is also concerned with chatter which occurs in the initial stage of cutting when interference between the previous and the present cut has not yet taken place.

The determination of the variation of the feed rate dr is quite simple. When superimposed on the nominal feed s_0 the vibration x is present the nominal rate of penetration $r = s_0\Omega$ becomes $(r + dr)$, where $dr = dx/dt$ and x is given by equation (1).

Substituting ds from equation (8) and $dr = dx/dt$ into equation (7) we obtain for the most general case

$$dP_x = k_1F_1(\alpha, \omega, \Omega, \mu)x + \left[k_1F_2(\alpha, \omega, \Omega, \mu) + \frac{K}{\Omega}\right]\frac{dx}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where

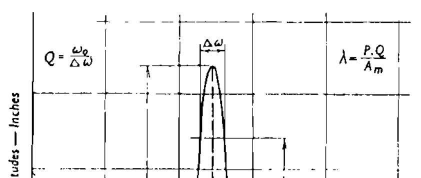
$$F_1 = \left[1 - \mu e^{-\alpha\frac{T}{m}}\left(\cos\omega\frac{T}{m} + \frac{\alpha}{\omega}\sin\omega\frac{T}{m}\right)\right]m\,;$$

$$F_2 = \left(\frac{1}{\omega}e^{-\alpha\frac{T}{m}}\sin\omega\frac{T}{m}\right)\mu m \quad . \quad . \quad . \quad . \quad (10)$$

Derivation of the Equation of Motion

Let us now direct our attention to the point where the cutting edge makes contact with the work piece and assume that a harmonic force of variable frequency (a vibration generator) is acting at these two points in the direction x, as shown on Fig. 1(*a*) to 1(*d*). By measuring the relative amplitudes between cutting edge and work piece and plotting them as a function of the exciting frequency a resonance curve of the type shown on Fig. 4 is obtained. The frequencies at which the amplitudes become a maximum correspond approximately to the natural frequencies of the system. It will be seen that under suitable conditions any one of these can be excited by regenerative chatter.

When the tool is highly flexible in the direction x and the structure is very stiff, then



The dynamic amplification factor Q and the dynamic stiffness λ are determined from the resonance curve (P = exciting force in pounds).

Fig. 4—Resonance curve of relative amplitudes between cutting edge and work piece in the direction x of Fig. 1

the resonance frequencies of Fig. 4 are easily calculated by giving due considerations to the end conditions. For instance, a lathe tool with a large overhang, a boring bar or an internal grinding wheel support, may be regarded as a cantilever clamped at one end. We shall assume that this cantilever is very stiff in the directions perpendicular to x. In this case, the natural frequencies of the cantilever are well separated and when vibrating in any one of them it can be represented by an equivalent single-degree-of-freedom system. Considering now one particular natural frequency ω_0 and its corresponding mode of vibration, the differential equation of the equivalent system can be written as

$$M\frac{d^2x}{dt^2} + c\frac{dx}{dt} + \lambda x = -dP_x \quad . \quad . \quad (11)$$

where M, c and λ are suitably chosen quantities for the particular mode of vibration. It is convenient to introduce non-dimensional quantities by putting

$$\lambda/M = \omega_0^2 \quad \text{and} \quad c/\lambda = 1/Q_e\omega_0 \quad . \quad . \quad (12)$$

where ω_0 is the natural frequency of the particular mode and Q_e is the effective amplification factor of the system. The structural amplification factor Q is determined from the resonance curve (Fig. 4) by dividing the natural frequency of the particular mode by the resonance curve width at $1/\sqrt{2}$ of the maximum amplitude. $2Q$ is the ratio of critical damping to actual damping. Q_e corresponds to the structural Q reduced slightly to account for the additional damping at the cutting edge ($Q_e < Q$). λ is the dynamic stiffness of the mode under consideration, which is also found from the resonance curve as

$$\lambda = \frac{\text{Exciting force} \times Q}{\text{Resonance amplitude}} \quad . \quad . \quad (13)$$

In general, the dynamic stiffness of the various modes differ from each other and from the static stiffness encountered by a static force which acts between work piece and cutting edge in the direction x.

By the introduction of the non-dimensional quantities of equation (12) and the consideration of the expression dP_x (equation (9)), the differential equation of the system can be written as

$$\frac{1}{\omega_0^2}\frac{d^2x}{dt^2} + \left(\frac{1}{Q_e\omega_0} + \frac{k_1}{\lambda}F_2 + \frac{K}{\lambda}\frac{1}{\Omega}\right)\frac{dx}{dt} + \left(1 + \frac{k_1}{\lambda}F_1\right)x = 0 \quad . \quad . \quad . \quad (14)$$

where F_1 and F_2 are given by equations (10).

It should be added that when the tool is very stiff in the direction x (for instance, in the case of drilling) its dynamic characteristics are determined mainly by its end conditions—that is, the dynamic characteristics of the structure. In this case, it is permissible to represent the system by an equivalent single-degree-of-freedom system as long as the natural frequencies of the structure are well separated and/or the damping of the corresponding modes is small. When neither of these conditions is satisfied the complexity of the problem is greatly increased, since the various modes become coupled and so the present method ceases to be directly applicable.

The Threshold of Stability Conditions

Substituting equation (1) into equation (14) we obtain two transcendental equations from which α and ω can be determined.

$$\left(\frac{\alpha}{\omega_0}\right)^2 - \left(\frac{\omega}{\omega_0}\right)^2 + \alpha\left[\frac{1}{Q_e\omega_0} + \frac{k_1}{\lambda}F_2 + \frac{K}{\lambda}\frac{1}{\Omega}\right] + \left(1 + \frac{k_1}{\lambda}F_1\right) = 0 \quad . \quad . \quad (15)$$

$$\frac{2\alpha}{\omega_0^2} + \frac{1}{Q_e\omega_0} + \frac{k_1}{\lambda}F_2 + \frac{K}{\lambda}\frac{1}{\Omega} = 0.$$

We are primarily concerned with the question whether a disturbance will build up ($\alpha > 0$) or decay ($\alpha < 0$), and so it is sufficient to consider the threshold of stability conditions ($\alpha = 0$), which are derived as follows

$$-\left(\frac{\omega}{\omega_0}\right)^2+1+\frac{k_1}{\lambda}m\left(1-\mu\sin\frac{2\pi}{m}\frac{\omega}{\Omega}\right)=0\quad .\quad(16)$$

$$\frac{1}{Q_e}+\frac{k_1}{\lambda}m\mu\frac{\omega_0}{\omega}\sin\frac{2\pi}{m}\frac{\omega}{\Omega}+\frac{K}{\lambda}\frac{\omega_0}{\Omega}=0\quad .\quad(17)$$

where the dimension of ω_0 and ω is c/sec.

The Stability Chart

Owing to the transcendental nature of equations (16) and (17) their direct practical application is most cumbersome and so they are conveniently presented in the form of a stability chart. This chart shows those rotational speeds (Ω/ω_0) at which the system is stable, unstable or at the threshold of stability, for a given structure (characterised by λ, ω_0 and Q), and a given tool, work piece material and machining condition (m, k_1, K, $Q_e=Q-\Delta Q$, and μ, &c.). The abscissa of this chart (Fig. 5 to Fig. 8) represents the non-dimensional rotational speed (Ω/ω_0) multiplied with the number of cutting edges m, where ω_0 is the natural frequency of the mode of vibration, the stability of which is being investigated. The ordinate corresponds to the effective $Q=Q_e$. The chart contains stable and unstable regions which are divided by boundary lines corresponding to the threshold of stability ($\alpha=0$). The method with which the boundary lines can be calculated from equations (16) and (17) has been described.[1] Some information concerning the stability chart can be obtained by the following considerations.

The Region of Unconditional Stability.—Equation (17) can be written

$$\frac{\omega}{\omega_0}B=-\frac{k_1}{\lambda}m\mu\sin\frac{2\pi}{m}\frac{\omega}{\Omega}\quad .\ .\quad(18)$$

where

$$B=\frac{1}{Q_e}+\frac{K}{\lambda}\frac{\omega_0}{\Omega}\quad .\ .\ .\ .\quad(19)$$

By squaring equation (18) and dividing it by $(\omega/\omega_0)^2$ from equation (16) we obtain after expansion

$$\cos^2\frac{2\pi}{m}\frac{\omega}{\Omega}-B^2\frac{1}{\mu m}\frac{\lambda}{k_1}\cos\frac{2\pi}{m}\frac{\omega}{\Omega}+B^2\frac{\lambda^2}{k_1^2}\frac{1}{m^2}\left(\frac{1}{\mu^2}+m\frac{k_1}{\lambda}\right)=0$$

$$.\ .\ .\ .\ .\ .\quad(20)$$

Regarding $B(Q_e)$ as a variable, there is some $Q_e=Q_m$ at which the roots of equation (20) become complex. This means that there is a minimum Q_e for which oscillation is still possible, but for $Q_e<Q_m$ the system is always stable. It can be shown that

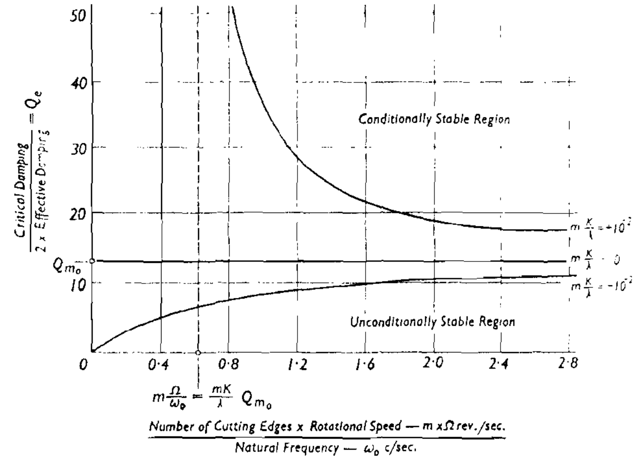
$$Q_m=\frac{1}{\sqrt{2\left[\mu^2\left(\frac{1}{\mu^2}+m\frac{k_1}{\lambda}\right)-\sqrt{1+2m\mu^2\frac{k_1}{\lambda}+\mu^4m^2\frac{k_1^2}{\lambda}\left(1-\frac{1}{\mu^2}\right)}\right]}-\frac{K}{\lambda}\frac{\omega_0}{\Omega}}$$

$$.\ .\ .\ .\ .\ .\quad(21)$$

The Q_m line divides the stability chart into the regions of unconditional stability $(Q_e<Q_m)$ and conditional stability $(Q_e>Q_m)$. Points lying in the region of unconditional stability correspond to chatter-free machining conditions. The region of conditional stability is divided into speed bands of stable and unstable points. The form of the Q_m line depends primarily on the parameters K, k_1 and λ.

Unconditional and Conditional Stability—Type-A Chatter.—We can distinguish two extreme cases depending on the overlap factor.

(*I*) $\mu=1\cdot0$.—The boundary line between the unconditionally and conditionally stable region is shown on Fig. 5 for three typical values of mK/λ. When $K=0$ the Q_m function becomes a constant Q_{m0}, indicated in the stability chart by a line parallel to the $m\Omega/\omega_0$ axis. When $K\neq0$ the Q_m line is a hyperbola which for $K>0$ has a vertical asymptote at $m\Omega/\omega_0=m(K/\lambda)Q_{m0}$ and a horizontal asymptote at Q_{m0}, approaching it from above, when $K<0$ the Q_m line is also asymptotic to Q_{m0} approaching from below and passing through the origin.



The shape of the Q_m line depends on the value of mk_1/λ (=0·04), μ (=1·0) and the value and sign of mK/λ.

Fig. 5—The Q_m line forms the boundary between the unconditionally and conditionally stable regions of the stability chart

(*a*) When $K=0$ it can be shown[1] that in the conditionally stable region the unstable bands are situated between vertical asymptotes which are found at the speeds

$$m\frac{\Omega}{\omega_0}=2\sqrt{1+(1+\mu)m\frac{k_1}{\lambda}}\ ;\ \sqrt{1+(1-\mu)m\frac{k_1}{\lambda}}\ ;$$

$$\tfrac{2}{3}\sqrt{1+(1+\mu)m\frac{k_1}{\lambda}}\ ;\ \&c.\quad .\ .\ .\quad(22)$$

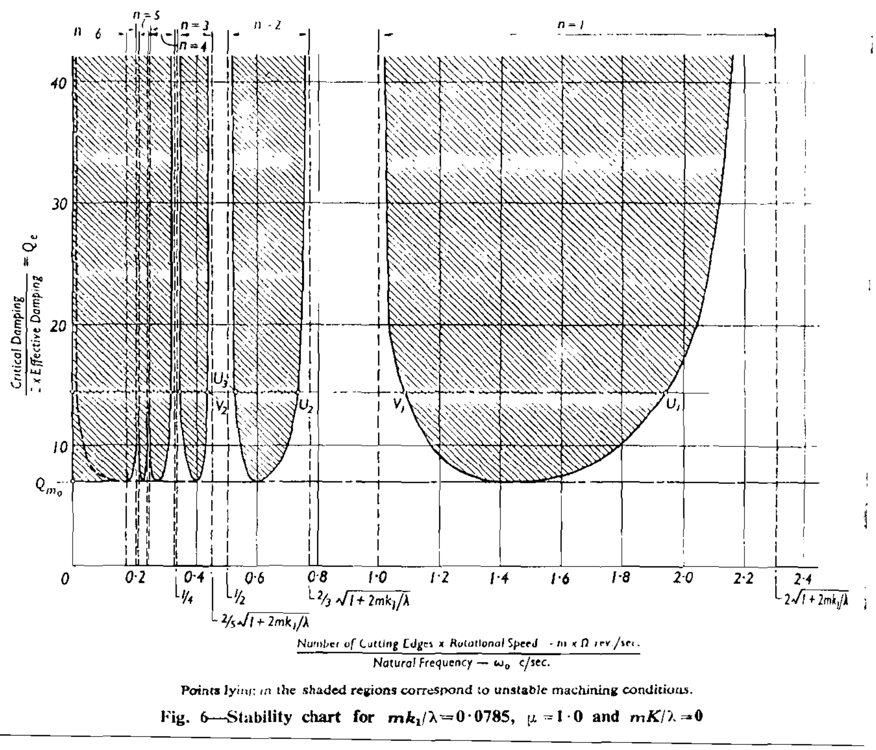
The system is always stable when $m\Omega/\omega_0$ is greater than $2\sqrt{1+(1+\mu)m(k_1/\lambda)}$. Below this limit the unstable speed band of *n*-th order is found between

$$\frac{1}{2n-1}2\sqrt{1+(1+\mu)m\frac{k_1}{\lambda}}>m\frac{\Omega}{\omega_0}>\frac{1}{2n}2\sqrt{1+(1-\mu)m\frac{k_1}{\lambda}}$$

$$.\ .\ .\ .\ .\quad(23)$$

Below a certain speed limit the unstable speed bands overlap.[1] This can happen even with the first and second unstable bands and then at relatively high values of Q_e we shall find a continuous unstable area. At Q_e values only slightly larger than Q_{m0} there will usually be stable speed bands between the lower order unstable bands. However, at the higher order unstable bands these dividing stable bands are so short that below a certain speed limit practically the whole area above the Q_{m0} line becomes unstable. This can be seen from Fig. 6, which shows the stability chart for $\mu=1\cdot0$ and $K=0$. Here overlapping commenced with the fourth band and extends to $m\Omega/\omega_0=0$, that is, zero rotational speed. However, at very low speeds the theory ceases to be valid owing to the elimination of the relief angle of the cutting tool,[1] and so we can expect that the overlapping unstable bands have a lower boundary which has the approximate shape indicated in the figure by the dotted line.

For finite values of Q_e, on the threshold of stability, the frequency of oscillation is always greater than the natural frequency of the mode under consideration. From equations (16) and (17) it can be shown that as the rotational speed sweeps through any one of

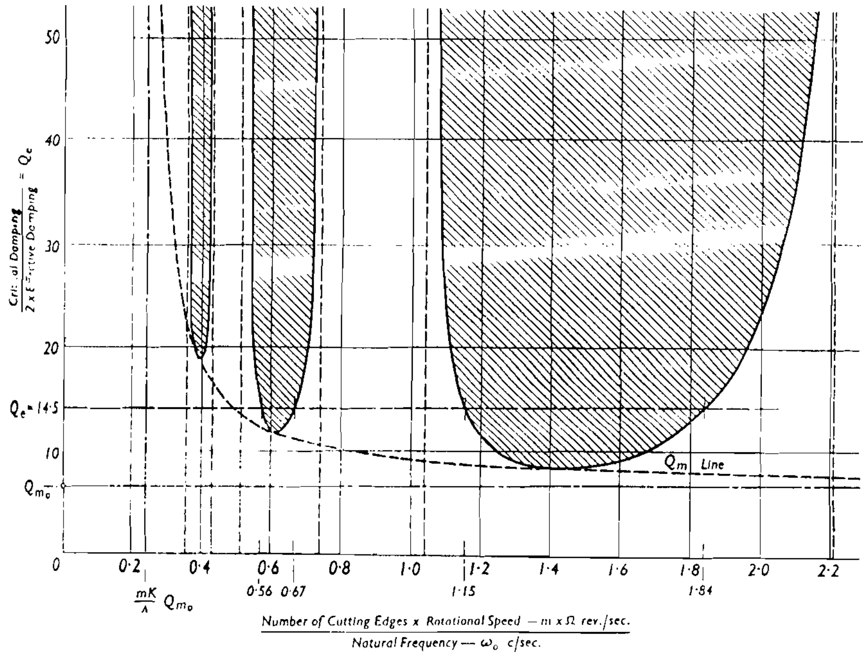


Points lying in the shaded regions correspond to unstable machining conditions.

Fig. 6—Stability chart for $mk_1/\lambda=0\cdot0785$, $\mu=1\cdot0$ and $mK/\lambda=0$

the unstable speed ranges in the direction $m\Omega/\omega_0 \rightarrow 0$ the frequency decreases from $\omega/\omega_0=\sqrt{\left[1+(1+\mu)m\frac{k_1}{\lambda}\right]}$ at $Q_e=\infty$, and becomes $\omega/\omega_0=\sqrt{\left[1+(1-\mu)m\frac{k_1}{\lambda}\right]}$ at $Q_e=\infty$, as $m\Omega/\omega_0$ enters a stable band again. The points lying on the right side of the threshold of stability curves which belong to a particular Q_e (points U_i in Fig. 6) have the same frequency of oscillation. The same is also true for points lying on the left side (points U'_i).

(*b*) When $K>0$ the higher order unstable bands are raised in the stability chart and are



When $K>0$, the lower order unstable regions are raised in the stability chart ($mk_1/\lambda=0{\cdot}0785$, $\mu=1{\cdot}0$ and $mK/\lambda=0{\cdot}0183$).

Fig. 7—Stability chart for $K>0$

less dangerous since they are excited only when the structural Q is very low and/or the stiffness is small. A stability chart of this type is shown on Fig. 7. It corresponds to conditions arising when drilling,[1] (as it has been verified experimentally), and it should also apply to grinding (Fig. 1(*d*)).

(*c*) When $K<0$ the higher order unstable bands are lowered and become consequently more dangerous since they can be excited also when the damping is large. According to the analysis of the drilling process given by the authors[1], stability charts of this type ought to occur in the case of drilling up predrilled holes and also when face milling and possibly when turning.

For $K\neq 0$ points of equal frequency will not lie on a line Q_e=constant, but on a hyperbola which is similar to the Q_m line. The important conclusions that $\omega/\omega_0>1$, and that by decreasing the drill speed the frequency of oscillation decreases, still hold. Both results have been verified for the case of drilling.

(II) $\mu=0$, that is, when there is no overlapping between successive cuts we get from equation (21),

$$Q_m=-\frac{\lambda}{K}\frac{\Omega}{\omega_0} \quad . \quad . \quad . \quad . \quad (24)$$

which is meaningful for positive values of Q_m. It is convenient to consider directly the damping coefficient of equation (14), which, in this case, takes the form

$$\frac{1}{Q_e\omega_0}+\frac{K}{\lambda}\frac{1}{\Omega}<0 \quad . \quad . \quad . \quad . \quad (25)$$

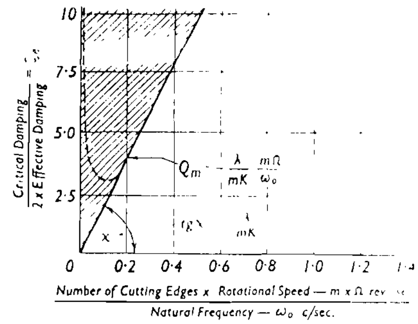
This expression must be negative for oscillations to occur. Since by definition $Q_e>0$ (otherwise the structure by itself would be dynamically unstable), the expression (25) can be negative only when $K<0$. Thus, when $K>0$ the system is unconditionally stable at all rotational speeds, no matter how small the structural damping may be.

For the case $K<0$ the stability chart is shown in Fig. 8. The Q_m line is a straight line passing through the origin. The region lying below the line is unconditionally stable and the area above is unconditionally unstable. Once more, the results are invalid in the vicinity of $m\Omega/\omega_0=0$, owing to the variation of the relief angle, and a boundary line of the approximate shape indicated by the dotted line can be expected.

As will be explained presently, in the case of drilling into solid material and when grinding $K>0$. For drilling up predrilled holes and for face milling it is likely that $K<0$, but $\mu=0$ only during the first $1/m$-th revolution of the tool. $\mu=0$ also occurs during the first revolution of the work piece when turning, and always when cutting a screw thread. In the last case chatter represented by Fig. 8 is likely to occur only when one side of the thread is being recut, since otherwise chatter in the direction x shown in Fig. 1(*c*) is probably not possible. However, the stability chart of Fig. 8 shows that chatter may occur even when the cutting speed is constant, and so the influence of a falling or rising cutting force versus cutting speed characteristic (Fig. 2(*b*)) does not enter at all into consideration.

From equation (16) we find that on the threshold of stability the chatter frequency is given by

$$\frac{\omega}{\omega_0}=\sqrt{1+m\frac{k_1}{\lambda}} \quad . \quad . \quad . \quad . \quad (26)$$



Chatter can occur only when $K<0$

Fig. 8—Stability chart for $\mu=0$

which is constant for all rotational speeds.

As had already been pointed out, the dynamic stiffness (equation (13)) of the various modes of vibration will, in general, differ from each other. Consequently, the stability charts of the various modes for the same cutting conditions take a different form. From the practical point of view it would be an advantage if one and the same stability chart would be applicable to all modes of vibration. This aim can be achieved under certain conditions by the introduction of the average dynamic stiffness λ' and the reduced effective Q (Q_e'), as will be shown in a subsequent publication.[2]

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[1] Tobias, S. A., and Fishwick, W., 1956, Vol. 170, No 6, *Proc. I.Mech.E.*, "The Vibrations of Radial Drilling Machines under Test and Working Conditions."

[2] Fishwick, W., and Tobias, S. A., 1955, Paper read at the 2nd FoKoMa, "The Effect of Flexible Supports on Machine Tool Chatter." (In the press.)

(To be continued)

British Standards Institution

All British Standard Specifications can be obtained from the Sales Department of the Institution at 2, Park Street, London, W.1.

BUILT-ON COOLANT PUMPS FOR INTERNAL COMBUSTION ENGINES (NOMINAL OUTPUT 1750-17,500 GALLONS PER HOUR AT 30FT MAXIMUM HEAD)

No. 2896 : 1957. Price 4s. 6d. This is one of a series of standards designed to secure a measure of standardisation of dimensions of components of internal combustion engines. Its main purpose is to facilitate the interchangeability of certain centrifugal coolant pumps and their components. It is also intended that wherever possible it should apply in whole or in part to all other forms of "built-on" pumps in the capacity range 1750-17,500 gallons per hour at 30ft maximum head. The standard specifies a range of four suction and discharge bore sizes — 1½in, 2in, 3in and 4in, and certain dimensions for suction and discharge flanges, suction and discharge stubs, the pump shaft in the immediate vicinity of the coolant seal, driving splines, the mounting flange and spigot and housing for coolant seals. It is envisaged that pumps embodying these features will be suitable for use on future industrial, locomotive or marine engines in the power range 150-2000 h.p. The standard, which is illustrated, contains an appendix which sets forth a number of useful recommendations on pump design.

FILLER RODS AND WIRES FOR INERT-GAS ARC WELDING: PART 1, GAS-SHIELDED TUNGSTEN ARC WELDING

No. 2901 : 1957. Price 6s. This standard contains requirements for the more commonly used ferrous and non-ferrous filler materials, although it has been thought advisable at the present time to omit requirements for filler rods and wires for mild steel this because of the current difficulties encountered in welding this material by the gas shielded tungsten arc process.

It is pointed out that although the rods and wires specified in this standard are all suitable for some form of gas shielding, certain rods and wires are not suitable for shielding with a particular gas, and purchasers should therefore ascertain from the supplier whether the rods or wires are suitable for a particular gas shielding.

any gain. Furthermore, if some of the oxides leave the nozzle in solid form, the exhaust will be very smoky.

Although liquid hydrogen appears so outstanding, it has one major disadvantage apart from its low boiling point. It is that its density is very low and consequently a very large capacity tank is required compared with that for a more conventional propellant giving the same total thrust. Not only does this affect the drag of a rocket adversely, but also the empty weight, particularly in view of the insulation problem. It is almost certain that at some stage in space rocket development, hydrogen will be used as fuel, but at the present time advances are more likely to be successful using hydrazine (N_2H_4) or ammonia (NH_3) as these give better overall propellant relative densities as indicated in Table II. In this connection it may be remarked that the solid propellants score heavily.

The more advanced oxidants are not attractive, apart from their potential performance, as they are either toxic, unsuitable or incompatible with most engineering materials. Pure ozone is extremely shock sensitive and will explode easily; similarly, liquid fluorine needs to be produced on the rocket firing site to avoid transport problems and the difficulties of controlling the toxic gas which constantly boils off.

In the motor itself, these advanced propellants raise problems of sealing, strength and corrosion. Most sealing materials become brittle at the low temperatures of liquid oxygen and other similar fluids, the design of joints must allow for thermal distortions and shock conditions where extreme changes in temperature occur, and lubrication of pump bearings and the operation of valves becomes difficult.

Reliability

Because the rocket vehicle and its propulsion system are designed with the barest minimum of safety margin, there are a number of risks of failure occurring before or during the launching. Apart from the risk of mechanical failure of a component, there are various functioning sequences which must operate correctly. For example, the pumps must be driven at the appropriate speed and propellant injection and ignition be correctly phased. It is probable that the ignition is the most critical event in the launching programme. Adequate energy must be supplied to the propellants as they enter the combustion chamber to ensure ignition without a delay of more than a few milliseconds. Otherwise there is the risk of propellant accumulation and a "hard" start in which the sudden ignition of a large quantity of propellant gives a high shock pressure with resultant damage. This ignition difficulty may be greater in the late stages of the multi-stage rocket when pressure and temperature conditions are lower than at ground level.

Generally, multi-stage rockets will have a much lower reliability than a single-stage unit. For example, if the chances of success are 90 per cent for a single-stage unit, the three-stage vehicle would have only a 0·9 × 0·9 × 0·9 = 73 per cent chance of success. The figure of 90 per cent used may well be on the high side for experimental rockets of the size involved in satellite launchings. This would be the stage reliability if the number of components involved was only 120 and the reliability of each one was 99·9 per cent.

It is clear, therefore, that the failure of the "Vanguard" rockets was not unlikely, and also that the Russian success points either to a large slice of luck or else to a very highly developed vehicle.

Theory of Regenerative Machine Tool Chatter*

By S. A. TOBIAS, D.Sc., Ph.D.,† and PROFESSOR W. FISHWICK, Ph.D.‡

(No. II—*Concluded from page* 203, *February* 7)

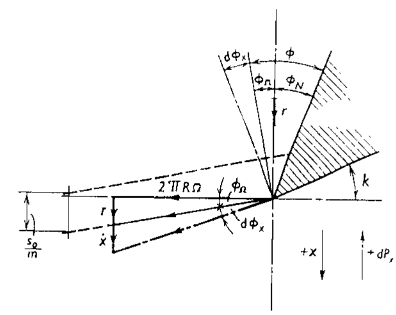
The Experimental Determination of the Factor k_1

Before we turn to the explanation of the use of the stability chart it will be convenient to discuss the physical significance and the method of determination of the parameters which affect its shape. The factors k_s and k_Ω appear in the work of previous investigators and so they do not need any further discussion. (k_Ω does not enter at all in the case of type A chatter.) Thus, we shall be concerned only with k_1 and $K=k_s-k_1$, which are entirely new and which may as a result be viewed with suspicion.

According to equation (7) the incremental thrust dP_x is required for the production of an alteration of the cutting conditions, specified by ds, dr and $d\Omega=0$ ($dv=0$). This force consists of two parts. The first part $k_1 . ds$ will be required for the removal of the additional chip thickness and the second part Kdr/Ω is necessary for the alteration of the feed rate. The term Kdr/Ω represents the difference between the total thrust increment $k_s dr/\Omega$, calculated as though dr was a steady state increase, and the actual thrust increment $k_1 dr/\Omega$, calculated for an independent variation of the feed rate by dr. This term can be interpreted as being essentially due to two physical effects, the penetration effect and the rake angle variation effect both dependent on the instantaneous feed rate.

Consider a cutting tool which is about to come into contact with the work piece. Before cutting can commence a finite pressure between the cutting edge and the material is required. To begin with, this pressure causes an elastic deformation at the point of contact, and after it has reached a certain level the work piece material commences to flow and the tool can penetrate. When the tool is cutting and it is required to leave its steady motion and penetrate deeper into the material, an additional thrust is necessary besides the thrust used for the removal of the increased chip thickness. This additional thrust is used for breaking the surface and to overcome the resistance due to a change of the angle of flow with respect to the cutting edge. The conditions arising are obviously of great complexity and are not yet fully clarified. However, it can be seen intuitively that the additional thrust is approximately proportional to dr and inversely proportional to Ω.[1]

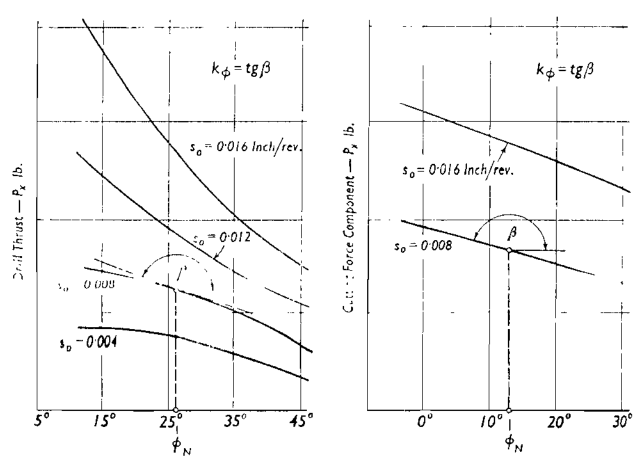
While the penetration effect causes an increase of the thrust for an increase of r, the rake angle effect results in its reduction. Fig. 9 shows a cross section of the cutting tool



A variation of the feed rate r by dr results in a rake angle variation of $d\Phi = dr/2\pi R\Omega$.

Fig. 9—Variation of the rake angle

of one of the cases shown in Fig. 1(*a*) to 1(*c*) in a plane which passes through the cutting velocity and the direction x. As can be seen from the figure, an increase of r by dr results in an increase of the effective rake angle Φ by $d\Phi = dr/2\pi R\Omega$, where R is a radius from the centre of rotation. This causes a change of the thrust increment by $k_\Phi . dr/2\pi R\Omega$, where $k_\Phi<0$. k_Φ is determined by machining with tools of various rake angles (Φ_N) under otherwise identical conditions and by plotting the thrust P_x as a function of Φ_N. The types of curves obtained for drilling are shown on Fig. 10(*a*) and those for turning on Fig. 10(*b*).



(*a*) drill thrust as a function of the helix angle; (*b*) cutting thrust component P_x as a function of the rake angle for turning.

Fig. 10—Determination of the factor k_ϕ

The determination of k_Φ from these curves is obvious from the figures and it is clear that $k_\Phi<0$.

Whether $K>0$ or $K<0$ depends on which of the two effects is the dominant. In the case of drilling into solid material the penetration effect is the dominant owing to the chisel edge which offers great resistance to an increase of the feed rate. The same is true for grinding since in that case one cannot speak properly of a rake angle and so in

* This article is an extended version of a paper read at the "Second Colloquium for Machine Tool Research and Design" (2.FoKoMa), held at the Technische Hochschule, München, 1955.

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both cases $K > 0$. When drilling up a pre-drilled hole the chisel edge is absent and according to the analysis given by the authors[1] we should find $K<0$. The same analysis ought to apply also to face milling and turning and so they should also have a $K<0$ or, at least, very small.

K and, to some extent, k_1 are dependent on a great many factors which affect the thrust. Among these it is worth while to mention the condition of the cutting edge (sharp or blunt), its geometry (shape and cutting angles, relief angle), the frictional conditions (cutting fluids), the work piece material, &c. It is one of the main advantages of the present theory that these factors do not appear explicitly and as a result the mathematical procedure is relatively simple. Their effect on the stability of machining is investigated by analysing their effect on the numerical value of k_s and k_1.

k_1 and so K can, in principle, be determined by both static and dynamic tests. In the case of static tests the experimental conditions are devised in such a way that the chip thickness s can be varied independently of the feed rate r. When shaping the chip thickness can be varied while keeping the rate of penetration constant ($r = 0$). By preforming the upper surface of the chip to be removed to vary in, say, a linear way and measuring P_x, we obtain $P_x(s)$ in a single test (for $r = 0$), and from this curve k_1 is determined by graphical differentiation. The same idea can be adopted to a lathe tool by shaping the end of a tube suitably and measuring P_x during one revolution. This experiment can be carried out for various feed rates and can also be adopted to grinding.

The dynamic determination of k_1 is most conveniently based on equation (9). The experimental conditions must be devised in such a way that both λ and Ω can be varied in small steps. λ is adjusted for chatter to occur in some rotational speed band. After that Ω is varied till the system is on the threshold of stability—that is, small oscillations of uniform amplitudes occur and so $\alpha = 0$. For this case equation (9) can be written

$$dP_x = k_1\left(1 - \mu \cos \frac{2\pi}{m}\frac{\omega}{\Omega}\right)mx + \left(k_1\frac{\mu m}{\omega} \sin \frac{2\pi}{m}\frac{\omega}{\Omega} + \frac{k_s - k_1}{\Omega}\right)\frac{dx}{dt} \qquad (27)$$

Assuming that k_s has previously been determined and so, since μ, m and $\Omega = 1/T$ are given, by measuring dP_x, ω, x and dx/dt the factor k_1 is easily calculated. Care must be taken that the experimental conditions correspond to the threshold of stability. If the conditions correspond to an unstable point of the stability chart the amplitudes will be large and non-linear effects enter which will affect the reliability of the measurements. If the adjustment of threshold of stability conditions proves to be impracticable, then records of the actual building up or decaying of the vibration amplitudes can also be used. These will yield α and in conjunction with equation (9) finally k_1 and so K.

The Use of the Stability Chart

It will be appreciated that one of the major advantages of the present theory lies in the fact that the conditions under which chatter can exist can be surveyed at a glance and that steps to be taken for its avoidance can be decided without taking recourse to tedious calculations of stability conditions. From the point of view of the application of the stability chart it is not even necessary to be familiar or to understand the underlying mathematical theory or the physical phenomena which produce chatter. In this sense the stability charts presented in this paper are similar to other charts used extensively in the design office or the workshop.

In general, the present theory recommends that when designing or using machine tools the dynamic stiffness λ for each mode between work piece and cutting edges should be kept as high as possible. This results in a raising of the Q_{m0} line in Fig. 5 to Fig. 7, as a result of which the area of the unconditionally stable region is enlarged. An identical result is achieved by decreasing k_1, which is dependent on the tool form and the work piece material. Similarly, K is also determined by these factors and an attempt to make K a large positive value is worth while, since this ensures freedom from chatter at low rotational speed. In those cases where the damping of the system is to some extent under control, large damping (low values of Q) obviously should be striven for since this also increases the stability of the system.

Consider now the application of the stability chart to a design problem. Since the natural frequencies and their corresponding damping Q of similar types of machines vary but little, the Q's of the various modes can initially be assumed to be known from previous tests. Let the various natural frequencies of such a machine be ω_{01}, ω_{02}, &c., and the corresponding Q's be Q_1, Q_2, &c. The designer is now given a series of stability charts corresponding to the type of machining under consideration ($K>0$ or $K<0$) which have been calculated for various values of the parameters k_1/λ and K/λ. From these he selects one chart for which the $Q_e = Q_1$, $= Q_2$, &c., lines (parallel to the $m\Omega/\omega_0$ axis in the charts) fall entirely in the unshaded region. He thus has fixed a value for k_1/λ. From the maximum required cut he can now find the maximum value of k_1, which is likely to occur and so he obtains λ, i.e. the required dynamic stiffness between work piece and cutting tool. This procedure may be modified in the case of special machine tools which have only one rotational speed and for which the most suitable speed is easily found from the stability chart. When designing entirely new machines the natural frequencies can be found by building and testing a model and using the results in conjunction with the stability chart for the determination of the most economical designs.

The machine tool user is advised by the stability chart in the proper choice of machining conditions (speeds, feeds, suitable tools and work piece materials and suitable foundations,[2] &c.). Let us assume that Fig. 7 represents the stability chart of a certain machining job which he is about to carry out. From previous tests he may know that in the particular machine a mode of vibration with ω_0 natural frequency and, say, $Q_e = 14 \cdot 5$ is liable to cause chatter. The $Q_e = 14 \cdot 5$ line in the stability chart intersects the shaded area (unstable regions) between $1 \cdot 84 > m\Omega/\omega_0 > 1 \cdot 15$ and $0 \cdot 67 > m\Omega/\omega_0 > 0 \cdot 56$. This means that if he wishes to avoid chatter the rotational speed must not fall in the speed bands $1 \cdot 84\ (\omega_0/m) > \Omega > 1 \cdot 15\ (\omega_0/m)$ and $0 \cdot 67\ (\omega_0/m) > \Omega > 0 \cdot 56\ (\omega_0/m)$. He can thus find the most suitable machining conditions. This procedure may be too cumbersome for most practical cases, but it will be worth while when the machine will carry out the same operation for a long period (for instance, in the case of transfer machines), since by the elimination of chatter tool life is greatly increased.

When buying a new machine the customer is able to compare the dynamic performance of competitive designs by simply comparing their stability charts determined for some standard machining conditions. Stability charts can also be used as a comparative measure for different designs of tools. When testing various tools on the same machine, Q_e and λ can be assumed to be kept constant. As a result the stability of each tool and the test machine will depend solely on k_1 and K. Thus, by determining k_1 and K for certain standardised conditions (machines, work piece material, &c.), the tool designer is able to give a direct measure of the chatter behaviour of his tools. He will be expected to design tools which have a small k_1 and large positive K values.

It should be added that the stability charts can also be used for the investigation of some special problems, for instance, the effect of flexible supports on the chatter behaviour of machine tool. A paper dealing with this problem is about to be published.[2]

From these remarks it will be clear that the problem of machine tool chatter will be solved economically only by a collaboration of the machine tool designer, the tool designer, the metallurgist and the machine tool user. There is no doubt that by designing machines of exceptionally high stiffness chatter can be eliminated by the sole efforts of the machine tool designer. The essential question is, however, whether this course is in fact economical and whether machines designed in this way will ever become competitive as long as the tools and the materials used are liable to chatter and insufficient care is taken by the user concerning the foundation. Since chatter is due to the interaction of a large number of factors, it is only to be expected that its economic solution will be achieved only by a collaboration of all concerned.

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[1] Tobias, S. A., and Fishwick, W., 1956, Vol 170, No 6, *Proc. I.Mech.E.*, "The Vibrations of Radial Drilling Machines under Test and Working Conditions."

[2] Fishwick, W., and Tobias, S. A., 1955. Paper read at the 2nd FoKoMa, "The Effect of Flexible Supports on Machine Tool Chatter." (In the press.)

University Scholarships

The firm of Guest, Keen and Nettlefolds, Ltd., is offering three university scholarships each year to boys who might not otherwise be able to accept a place at a university. This scheme is in line with those recently instituted by other companies and engineering organisations and is designed to help boys whose parents' incomes are such that their state scholarships or local education authority grants are drastically reduced by the operation of a means test. The firm will pay the students' fees to the university or college in the normal way, but the students will rank as employees of the company and their maintenance and costs will be paid as a salary, subject only to deductions of income tax and National Health Insurance.

The students, of whom two each year must be engineers, will be selected by a special board. G.K.N. students will work during their long vacations either with group companies in this country or with recognised companies on the Continent, and will be paid an additional salary. The time spent on this approved workshop training will be deducted from the two years required for the full graduate apprenticeship, which will be served with the company. Application forms may be obtained from the Group Personnel Officer, Guest, Keen and Nettlefolds, Ltd., Shell-Mex House, Strand, W.C.2. The closing date for this year's applications is April 30.