The Stability of the Machine Tool Against Self-Excited Vibration in Machining

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I. Introduction

This paper deals very briefly with the fundamental research carried out in VUOSO in the field of self-excited vibration occurring in machining. More detailed information about this work can be found in the literature [1-3]. The work was aimed at determining the influence of the characteristics of the machine tool on the occurrence of self-excited vibration in order to be able to get higher stability of machine tools at minimal weight. The theory of self-excited vibration presented here explains all basic characteristics of this phenomenon, and some of the effects of changes in the machine tool on the vibration. Among these are, for instance, cases in which changes of rigidity of the machine in different directions cause different degrees of stability, or cases in which a change of the orientation of cutting in the machine (*e.g.*, clamping of a lathe tool downwards instead of upwards and reversing the direction of workpiece rotation) strongly influences the vibration. A classic example of such a change of orientation is the case of the boring bar with rectangular cross-section, described in Section VI.

The entire work has been oriented toward the analysis of the qualities of the machine and, consequently, the machine in our theory has been considered as a vibratory system with any number n of degrees of freedom, in contrast to the work of most other authors who have studied self-excited vibration in machining and have paid attention mainly to the analysis of the cutting process. In the mathematical treatment they have simplified the machine to a system of only one degree of freedom. On the basis of our theory a simple graphical method for the calculation of the limit of stability has been worked out, which is described in Section IV.

In contrast to the work of other authors in the mathematical explanation of self-excited vibration, only the dependence of the cutting force on the relative position of tool and work-piece has been used and it was found unnecessary to take the cutting force as a function of the velocity of the relative movement into account. As far as the authors know, this type of self-excited vibration, based purely on a "positional feed-back" concept has never been described before in the theory of mechanical vibrations. The theory, however, does not lack of generality and, as shown in [3], all the basic equations remain valid for other more complicated and more general relations between the cutting force and vibration.

II. Basic Scheme. Cutting Process as a Measure of the Stability of the Machine

The process of self-excited vibration in machining can be diagrammatically described by a closed-loop system in Fig. 1. It shows in (1) the cutting process, for which the input value is the relative vibration y of the tool and the work-piece and the output value is the variable component p of the cutting force. The block (2) represents the machine, on which the cutting force p is acting, causing vibration y. The number (3) expresses the relative orientation of the cutting process and the machine, *i.e.* mainly the position of the cutting process

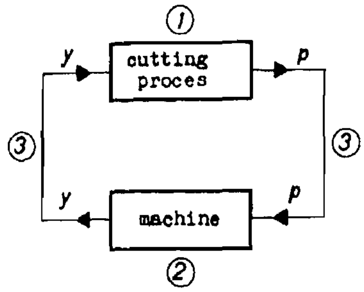

FIGURE 1

in relation to the vibratory system of the machine. First, blocks (2) and (3) of the diagram will be treated. Next, some more fundamental remarks will be made on the process of self-excited vibration and the cutting process.

Self-excited vibration in machining has the basic quality of all self-exciting processes: for certain values of the parameters of the whole system (parameters of parts (1), (2) and (3)) vibration does not develop at all and one has stable machining; at other values of the parameters vibration occurs and grows, resulting in unstable machining. In practice the system is always nonlinear. Therefore the amplitude of vibration grows only to a finite value. However, we are not interested in this final value of the amplitude. We are concerned only with the question of stability.

The cutting process is characterized by a number of parameters, such as cutting conditions, material of the workpiece, material of the tool, cutting speed v, tool angles α and γ, thickness of chip a (proportional to the feed), width of chip b (proportional to the depth of cut) (see Fig. 2). By changing the cutting conditions it is possible on a given machine to move from stable to unstable machining. The conditions at which the vibration begins to occur are called "limit conditions" of stable machining.

In Fig. 2, for the sake of simplicity orthogonal cutting is illustrated. It is obvious that the magnitude of the coupling between the relative vibration of tool and workpiece and the cutting force is proportional to the width of chip b (the whole cutting process is performed identically in each unit of the length of the cutting edge). Actually the influence of the width of cut b on the vibration, is the most important factor. At sufficiently small width of cut b the machining is stable. By enlarging b, all other conditions remaining constant, a certain value of b can always be reached, at which the process converts to an unstable machining condition. This is the *limit width of cut* b_{lim}.

In studying the influence of the vibratory system of the machine on self-excited vibration, it is the value b_{lim}, which will be used as a measure of the degree of stability of the machine. The stability of various machines (or of various modifications of a given machine) is compared by comparing the values b_{lim} determined in individual cases, all other cutting conditions being identical.

III. Principal Explanation of Self-Excited Vibrations in Machining

The condition for the development of self-excited vibrations in a linear vibratory system is that the force acting on the system (in our case the alternating component of the cutting force) has a component 90 degree phase shifted to the vibration (this component is 180 degree phase shifted to the damping force). Only then can the process deliver to the vibratory system the energy necessary to balance the damping losses.

Generally it can be assumed that the relative vibration of tool and workpiece is performed in 3-dimensional space and has components x, y, z in the directions X, Y, Z (see Fig. 2). It can be also assumed that the alternating component p of the cutting force depends on all vibration components x, y, z and also on its velocities $\dot{x}$, $\dot{y}$, $\dot{z}$. If we suppose the dependence of p on $\dot{x}$ or $\dot{y}$ or $\dot{z}$, the necessary phase shift between the force and the vibration occurs already in the cutting process. This supposition of the *velocity relation* has been used in early studies on self-excited vibration in machining, for instance by Arnold [5]. Later a number of other authors used this explanation, *e.g.*, Sokolovskij [6], Stefaniak [7], Saljé [8], Sadowy [9], Tobias [10], Shaw-Hölken [11], Shaw-Sanghani [12]. Various authors give various physical reasons for the *velocity relation.* This principle is treated in Section IIIA. It is not incorporated in our theory.

Some authors, as Stefaniak, Tobias, Hahn use the *principle of reproduction*, which does not suppose the dependence of the cutting force on the velocity of vibration. It is based on the special nature of machining, in which the tool cuts in a surface which it has previously cut. This principle is incorporated also in our theory and is explained in Section IIIC.

Another principle, which does not depend on the cutting force but on the vibratory displacement, is the principle of *positional coupling*, and is explained in Section IIIB. This principle, published for the first time by the

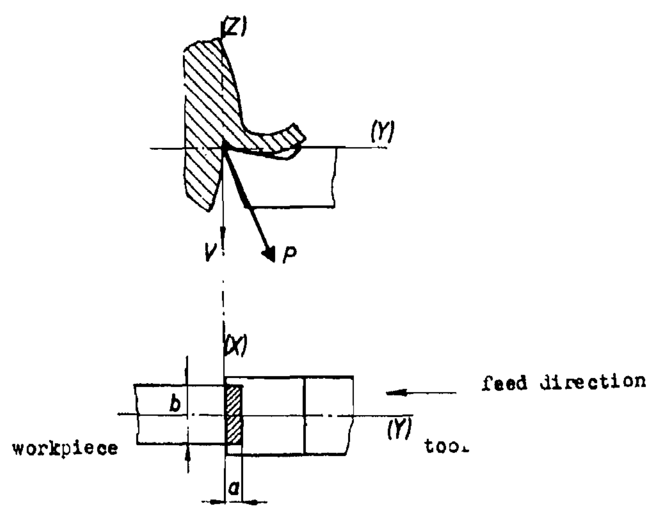

FIGURE 2

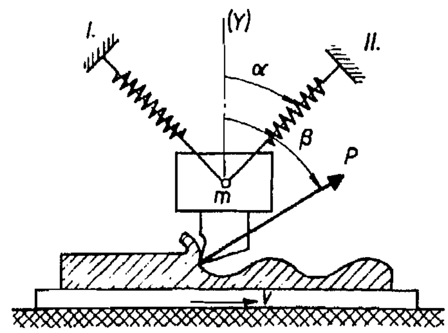

FIGURE 3

authors of this paper, was developed as the basis of our theory.

A. "Velocity Relation" for the Cutting Force. This principle can be best shown when a vibratory system with one degree of freedom and a linear relation between the alternating force p and the velocity $\dot{x}$ is assumed.

In the equations of motion of the system

$$
\begin{aligned}
m\ddot{x} + c_d\dot{x} - c_v\dot{x} + kx &= 0 \\
m\ddot{x} + (c_d - c_v)\dot{x} + kx &= 0
\end{aligned}
\qquad (1)
$$

a difference between the damping and velocity coefficients occurs. If this difference $(c_d - c_v) < 0$, the solution of the equation represents an unstable motion.

This principle does not explain a number of observed characteristic facts, the existence of the cutting force component depending on the vibration velocity has not been sufficiently quantitatively ascertained and it is obvious that the self-exciting energy which could originate in this way is very small in comparison with sources explained in Sections IIIB and C. Therefore, we do not apply this principle in our theory.

B. "Positional Coupling" Principle. In this principle a vibratory system with at least two degrees of freedom

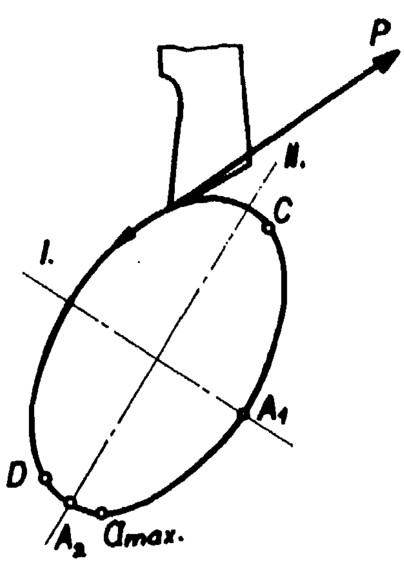

FIGURE 4

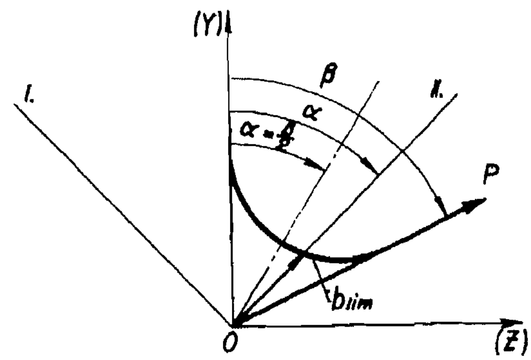

FIGURE 5

in two different directions must be assumed. A simple case of such a system is shown in Fig. 3.

The relation between the cutting force and the vibration can have a very simple form. Let us suppose (Fig. 2) that the change of the cutting force is influenced only by the vibration component in the direction (Y), which changes only the thickness of cut. The cutting force consists of a constant average value P_a, which depends on the average cut thickness a and on other cutting conditions, and of a vibratory component p, which is proportional to the vibratory displacement y, *i.e.*, to the change of cut thickness a.

$$p = -ry \qquad (2)$$

The value of the *coupling coefficient* r depends on the cutting conditions and is proportional to the cut width b.

Corresponding to Fig. 3, the tool is fixed to the mass m of the vibratory system with natural modes in directions I and II. Natural frequencies and stiffnesses of the system in the directions I and II are Ω_1, k_1, and Ω_2, k_2, respectively. The system is damped although the dampers are not drawn in the diagram.

Let us suppose that the system vibrates with frequency ω in both directions. The vibration in one direction will be generally phase shifted in relation to the vibration in the other direction. Then the mass and the cutting edge will performan an elliptical movement as shown in Fig. 4. Experimentally such a movement of the tool in self-excited vibration in machining has been observed by Hahn [13].

Maximal displacement in the direction I (into the cut) occurs at the moment when the tool point is at the point A_1, maximal displacement in the direction II occurs at the point A_2, whereas the maximal value of the cutting force occurs at the moment when the tool point reaches point a_{max}. It is evident that the alternating force component may have a phase with respect to the vibrations I and II. Mathematical treatment of this case (see [1]) shows that in order for self-excited vibration to occur two conditions must be fullfilled: the width of cut must be greater than the limit width b_{lim}, the direction of the mode with lower natural frequency must fall between the directions (Y) and P. (For the case of Fig. 3 this condition is true if $k_1 > k_2$. If it is not so, the system is absolutely stable (for all values of b).

The value of the limit width of cut $b_{\lim}$ depends on the stiffnesses and damping of the system and also on its directional orientation. This is illustrated in Fig. 5 in which the calculated limit width $b_{\lim}$ is drawn as a function of the angular position of the direction II of the mode with lower natural frequency. The curve $b_{\lim}$ separates the stable and unstable regions. Self-excited vibration develops at the lowest value of $b_{\lim}$ if $\alpha = \beta/2$.

The origin of the self-exciting energy in the positional coupling principle can be explained in the following way. The direction of movement along the elliptical path is supposed as shown by the arrow in Fig. 4. During the first half of the tool movement, from point C to D, the tool moves against the cutting force and energy is dissipated from the vibratory system. During the second half of movement, from D to C, the projection of the cutting force has the same direction as the movement, and energy is delivered to the vibratory system. Because in the second half of the movement the tool moves at a greater depth then in the first half the cutting force is greater, and therefore the work performed is greater. There occurs a surplus of energy which can supply the damping losses.

C. Principle of Reproduction. In all actual machining cases the tool cuts in such a way that, once vibration has occurred, the waviness of the surface generated during the preceding revolution modulates the cross section of the cut. In Fig. 6 a simple case is illustrated. On the surface to be cut there is a waviness Y_0. The tool is attached to the mass m of a simple vibratory system with stiffness k and damping d. The system vibrates and creates a new wavy surface Y. The peaks of both successions of waves are shifted by the value ϕ.

Let us again suppose that the cutting force depends only on the cross section of the cut, being larger at greater momentary thickness of the cut and being in phase with it. The momentary thickness value equals the average thickness a_a plus the difference between the ordinate y_0 of the original waviness with the amplitude Y_0 and the ordinate y of the developing waviness with the amplitude Y.

$$a = a_0 + y_0 - y \ . \tag{3}$$

The moment t_m, in which the momentary thickness a and correspondingly also the alternating force component is at maximum, is phase shifted in relation to the instant at which y reaches its maximum. The phase shift ϕ adjusts itself so that instability occurs at the least value $b_{\lim}$ possible.

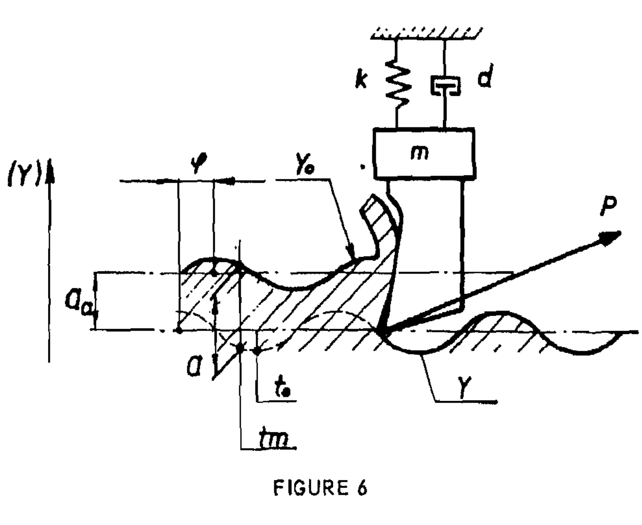

FIGURE 6

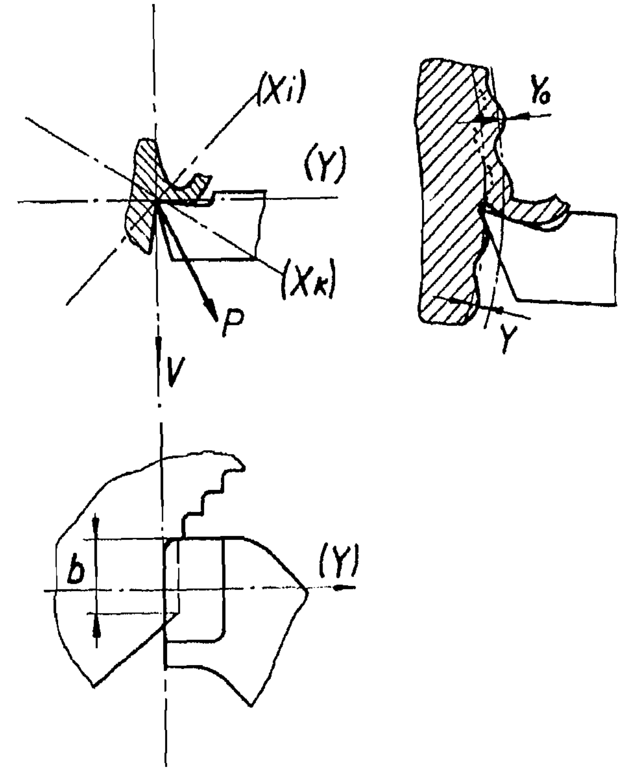

FIGURE 7

IV. Calculation of the Limit of Stability

In our theory the simplest supposition is accepted, that the instantaneous value of the cutting force depends only on the instantaneous cross section of cut. Therefore only the principles of positional coupling and of reproduction come into application. The dependence of the cutting force on vibration is expressed by the following conditions.

1) According to Fig. 7 the relative vibration of tool and workpiece is performed simultaneously in several directions, e.g., (X_i), (X_k), with different phase relations (the resulting relative movement being an ellipse), the cutting force depends only on the projection of the displacement in the direction (Y).

2) The amplitude P of the force is proportional to the amplitude of the change of the cut thickness. The tool cuts a surface, the waviness of which has the amplitude

Y_0. The vibration has the amplitude Y. The force amplitude is then expressed by

$$P = - r(Y - Y_0). \qquad (4)$$

3) The coupling coefficient r is a real positive number. Its value depends on the cutting conditions, mainly on the cut width b. The coefficient r is approximately proportional to b.

The vibratory system of the machine is actually a system with an infinite number of degrees of freedom with continuously distributed mass and elasticity. Although the mass and elasticity may be nonuniformly distributed, the system has a finite number n of distinct degrees of freedom. The system of the machine, which is three-dimensional, is diagrammatically shown in Fig. 8. To each of the degrees of freedom there corresponds

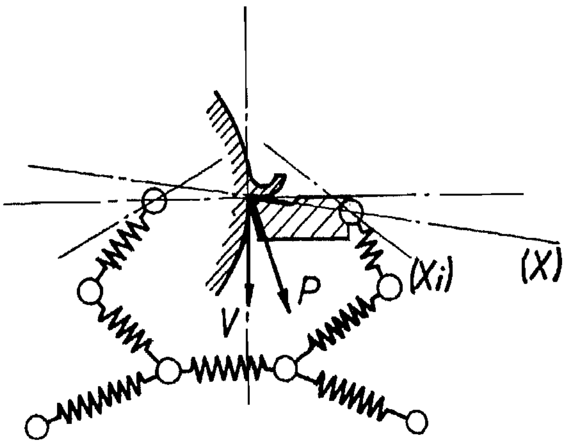

FIGURE 8

a natural frequency Ω_i, a damping coefficient δ_i, and at the point of the tool a natural relative stiffness k_i and a relative direction of natural vibration (X_i). To each of the modes of natural vibrations corresponds a mode shape, given by the relative amplitudes of individual points of the machine. An example of four important modes of natural vibrations of a milling machine is shown in Fig. 9.

The direction of natural vibrations (X_i) of a mode is very important in regard to the role which this mode will play in the process of self-excited vibration. This will be explained by a simplified example, in which the direction (X_i) lies in the plane (Y, v), Fig. 10. Let us denote α_i as the angle between the normal Y and the direction (X_i) and β the angle between Y and the cutting force P.

An alternating force with the amplitude P and frequency ω, forces vibration in the direction (X_i) only by its projection $P \cos(\alpha_i - \beta)$, so that the vibration amplitude X_i is

$$X_i = P \cos(\alpha_i - \beta) \frac{1}{k_i} \frac{\Omega_i^2}{\Omega_i^2 - \omega^2 + 2j\delta_i\omega} \qquad (5)$$

where $j = \sqrt{-1}$.

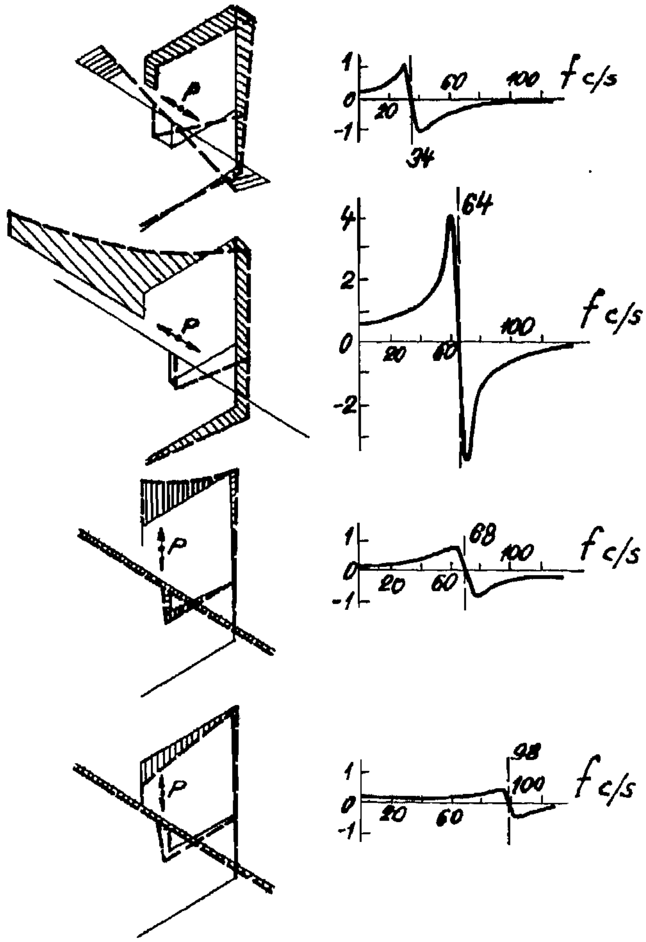

FIGURE 9

In the process of self-excited vibration only the projection y_i of the vibration x_i in the direction (Y) is applied. Its amplitude Y_i will be

$$Y_i = P \cos(\alpha_i - \beta) \cos\alpha_i \frac{1}{k_i} \frac{\Omega_i^2}{\Omega_i^2 - \omega^2 + 2j\delta_i\omega} \qquad (6)$$

The expression

$$u_i = \cos(\alpha_i - \beta) \cos\alpha_i \qquad (7)$$

which we call the *directional factor* (and the generalization of which for a three-dimensional case can easily be imagined) influences strongly the value of the i-th natural mode.

Two extreme cases should be mentioned, denoted in Fig. 9 as $(X_1) \perp (P)$ and $(X_2) \equiv (v) \perp (Y)$, for which both $u_i = 0$. The natural modes with these directions have no validity in the process of self-excitation, even though they had the smallest stiffness. The mode with the direction (X_1) has no value because the cutting force has no projection in it and does not force any vibration of it. The mode (X_2) has no value because its vibration does not deliver any component into the direction (Y) and does not influence therefore the chip cross section.

The alternating force $P(\omega)$ acts simultaneously on all n modes of natural vibrations. The resulting amplitude of

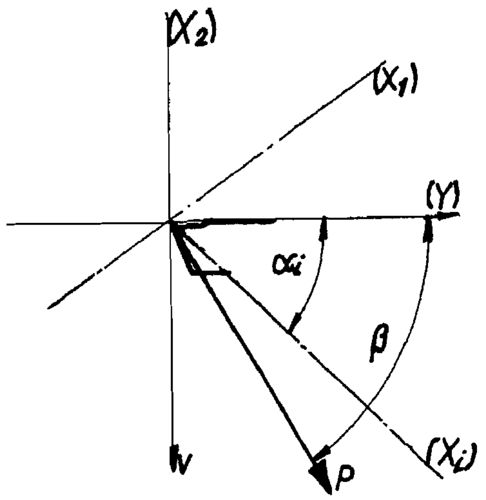

FIGURE 10

vibration in the direction (Y) is given by the sum of the projections of the amplitudes of all individual modes.

$$Y = \sum_{i=1}^{n} Y_i = P \sum_{i=1}^{n} \frac{u_i}{k_i} \frac{\Omega_i^2}{\Omega_i^2 - \omega^2 + 2j\delta_i\omega} \quad . \tag{8}$$

Let us denote

$$F(\omega) = \sum_{i=1}^{n} \frac{u_i}{k_i} \frac{\Omega_i^2}{\Omega_i^2 - \omega^2 + 2j\delta_i\omega} \quad , \tag{9}$$

so that (8) can be briefly written as

$$Y = P \cdot F(\omega) \quad . \tag{10}$$

To establish the condition for the limit of stability, let us eliminate the variable P from (4) and (10). Then we get after some modification the quotient q.

$$q = \frac{Y}{Y_0} = \frac{F(\omega)}{F(\omega) + \frac{1}{R}} \quad . \tag{11}$$

The function $F(\omega)$ is complex and accordingly the quotient q is also complex. Its absolute value $|q|$ signifies the ratio of the absolute amplitudes of waves Y and Y_0. If $|q| < 1$, the depth of waves on a cut surface will be smaller than in the preceding cut. The waviness will subsequently diminish and the system is stable. On the contrary, for $|q| > 1$ we get an unstable case. The value $|q| = 1$ corresponds to the limit of stability.

Let us express now the relation between the coefficient r representing the cutting process and the qualities of the vibratory system represented by the function $F(\omega)$, at the limit of stability. Let us resolve $F(\omega)$ into its real and imaginary parts:

$$F(\omega) = G(\omega) + jH(\omega) \quad .$$

At the limit of stability there will be

$$|q| = 1 = \left| \frac{G + jH}{G + \frac{1}{r} + jK} \right| \quad . \tag{12}$$

After some modification we get the condition for the value of r on the limit of stability:

$$-\frac{1}{2r_{\lim}} = G(\omega_{\lim}) \quad . \tag{13}$$

Practical use of (13) will be shown by the example of the simple vibratory system in Fig. 3. The function $G(\omega)$, the real part of $F(\omega)$, has for a one degree-of-freedom system in the following form

$$G(\omega) = \frac{u}{k} \frac{\Omega^2 - \omega^2}{(\Omega^2 - \omega^2)^2 + 4\delta^2\omega^2} \quad , \tag{14}$$

which is graphically shown in Fig. 11. The limit of stability is given by the minimum point of $G(\omega)$. The corresponding ω value is the $\omega_{\lim}$. From the diagram the value $1/2r_{\lim}$ can be directly obtained from which the value $r_{\lim}$ can easily be determined. For all other values $r < r_{\lim}$, $|q| < 1$ and the system is stable. For all values $r > r_{\lim}$ always a value ω can be found, for which $|q| > 1$ and there occurs instability.

A numerical calculation of $r_{\lim}$ from (13) would be practically impossible. Therefore the graphical method shown in Fig. 11 has been proposed, which is easy even for very complicated vibratory systems. According to this method the curves $G_i(\omega)$ for all individual natural modes are drawn and algebraically added together. The minimum point of the summing curve determines the values $r_{\lim}$ and $\omega_{\lim}$.

V. Example of the Calculation of a Milling Machine

The practical application of the stability calculation will be explained by the example of a milling machine. In the first part of the work the limit cutting conditions are determined by machining tests for various modifications of the machine. For the calculation, the case of milling with cylindrical cutter is chosen, in which the overarm is not joined with the knee. This case has been found as one of the most sensitive to self-excited vibrations.

In the second stage the machine is artificially vibrated by means of a vibrator which is placed in between tool and workpiece. The artificial alternating force acts, in one case vertically and in another horizontally with variable frequency. Resonance characteristics of relative vibration and mode shapes of natural vibrations are measured. From the resonance characteristics the values Ω_i, k_i, δ_i and the directions (X_i) are calculated.

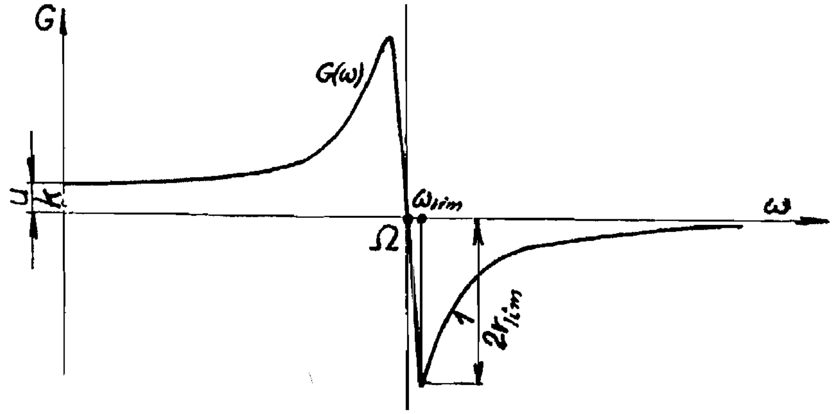

FIGURE 11

The values obtained are assembled in Table I.

TABLE I

Mode No.	Frequency cps	Natural Stiffness kp/μm	Relative Damping	Direction of Vibration
1	34	4	0,06	horizontal
2	64	2	0,035	horizontal
3	68	4	0,07	vertical
4	98	9	0,055	vertical

From these values the corresponding curves $G_i(\omega)$ are constructed as shown in Fig. 9. (There the G/u curves are given.)

The graphical solution of (13) is performed for various cases of up and down milling, which differ in the directions of the normal (Y) to the cut surface and of the cutting force P and consequently also in the values of the directional factors u_i for individual natural modes of vibration. In Fig. 12 the examples of the solution for one case of up milling (see Fig. 12(a)) and for one case of down milling (Fig. 12(b)) are given. In the left-hand side of the figure the position of the tool and directions (Y) and (P) are diagrammatically drawn for a depth of cut of 3 mm; in the middle the value of the directional factor u_i is graphically deduced. In the right hand side the graphical calculations of the values $r_{\lim}$ (which correspond to the limit widths of the cut) are given.

The results, which are in agreement with the results of machining tests, incorporate some interesting notions. In the first case, where $r_{\lim} = 0{,}66$ kp/μm and $\omega_{\lim} = 69$ cps, the limit of stability is mostly influenced by the mode with natural frequency 64 cps. The other modes influence very slightly the summing curve G at its minimum point. In the second case, because of the change of directional orientation, the factor u_2 is smaller and negative and the factor u_3 is greater than in the first case. The value of G_2 in the neighborhood of its minimum is compensated by G_3 and the minimum point of the summing curve (interrupted line) is shifted to the frequency 104 cps. The corresponding value $r_{\lim} = 1{,}28$ kp/μm is approximately two times higher as in the preceding case.

In the machining tests the limit of stability was attained in the first case at a width of milled surface of 25 mm and a frequency of vibration of 70 cps. In the second case a limit width of 50 mm and vibration frequency of 104 Hz were found. The agreement between the experiment and the calculation is very good. Thus the calculation is verified and it is possible to utilize all conclusions which can be taken from the analysis of Figs. 12(a) and 12(b). These indicate, for example, that first of all the stiffness of the mode 64 cps should be increased. By considering the mode shapes shown in Fig. 9, corresponding design changes of the machine can be proposed.

VI. Example of the Boring Bar

A characteristic example of the utilization of the change of directional orientation for the improvement of stability is the case of the boring bar. On a horizontal boring machine the boring head with a single tool is attached to the end of the boring bar, which has two opposite slots along the whole length. The cross-section of the bar is shown in Fig. 13.

If the small influence of the other parts of the machine (bed, column, table) on the qualities of the vibratory system at the point of the tool is neglected, then the vibratory system is reduced approximately to the mass of the boring tools and the spring represented by the bar. The slots in the bar cause the stiffness and frequency in direction *I* to be lower than in direction *II*. As the tool is fixed with the bar and rotates with it, the relative positions of directions *I*, *II* and the direction of the cutting force P and of the normal to the cut surface (Y) remain constant. If we change the clamping of the tool so that its angular position (see Fig. 14) with respect to directions *I* and *II* varies, the limit width of the cut varies also.

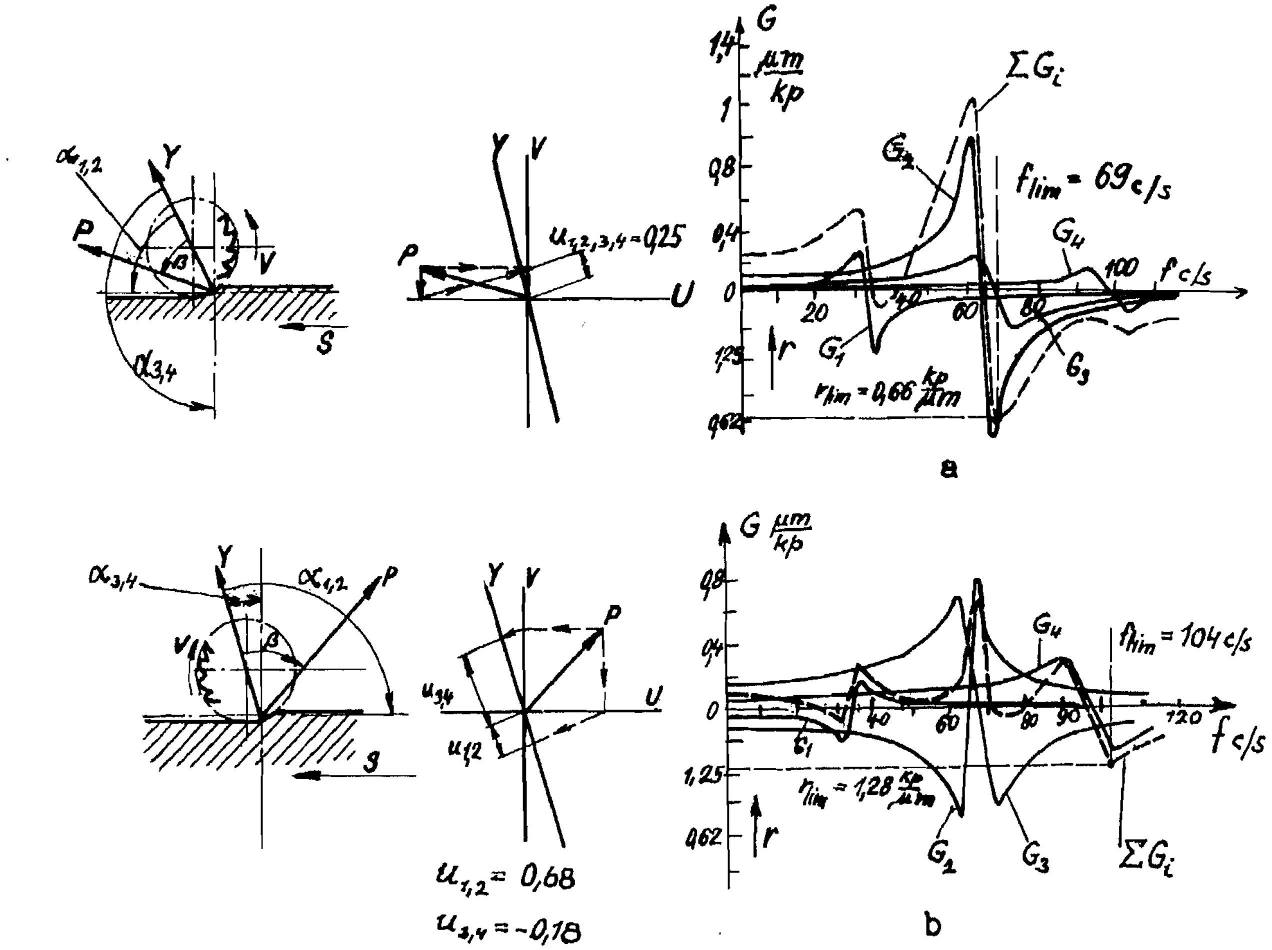

FIGURE 12

For a certain length of the overhang of the bar natural frequencies of 60 cps in direction *I*, and 75 cps in direction *II* have been measured. For an approximate stability calculation the system was simplified to two degrees of freedom with the two given frequencies.

The calculated values of r_{lim} are drawn in the polar diagram Fig. 15 in relation to the angle α (Fig. 14). The limit curve in Fig. 15 is symmetrical about the axis (Y). During the change of the orientation in the range of 360 degrees, the limit curve has four maxima and four minima. The ratio of r_{lim} maximum to r_{lim} minimum is roughly 4 : 1.

In the machining tests coincidence of the optimal and worst positions of the tool with the positions calculated was found. The ratio of r_{lim} max to r_{lim} min was found to be approximately 6 : 1. The quantitative difference of this ratio calculated and experimentally observed is

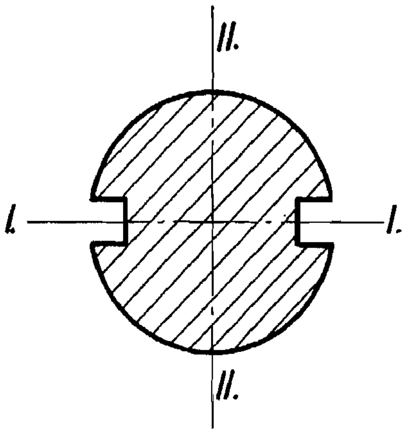

FIGURE 13

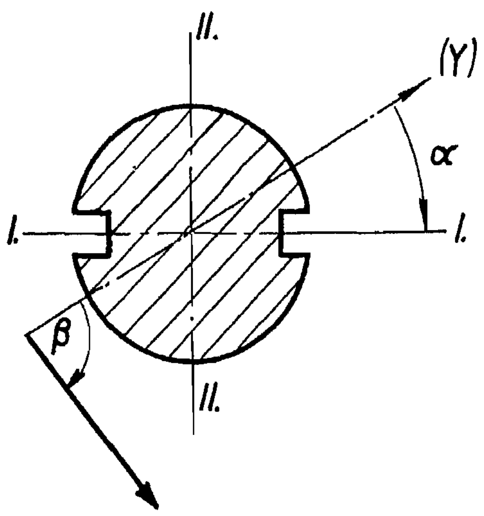

FIGURE 14

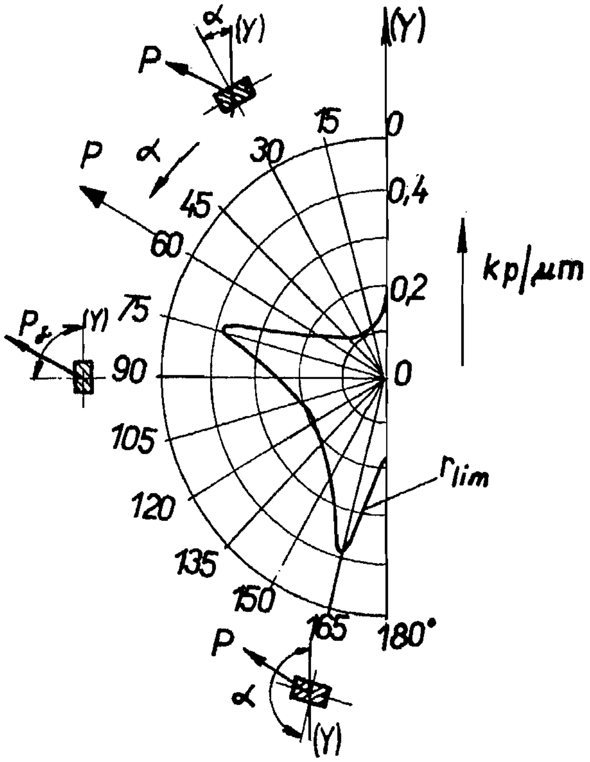

FIGURE 15

explained by the simplification of the system in the calculation. If the actual values of the system are taken a better agreement of the test results and the calculation can be obtained, as has been done, for instance, in the experiments of Péters [14]. In Fig. 16 reprinted from [14] the limit width measured in the tests (full line) and calculated (interrupted line) are given. The calculation of Péters has been performed by a graphical method which differs slightly from out method, but is based on identical assumptions. The results obtained are the same.

VII. Conclusion

The authors have applied the theory and method of calculation here described to the analysis of quite a number of various types of machine tools. Experience shows that under simplifying conditions on which the described theory and calculation are based, the accuracy of the description of the process of self-excited vibration from the point of view of the role of the machine in this process is sufficiently high and there is no practical reason to complicate the theory in the area of the cutting process for the given purpose.

This is true for all basic kinds of machining such as turning, boring, planing, milling, with the exception of grinding (both external and internal). Grinding differs from all other mentioned kinds of machining in two aspects: in all actual grinding cases the value of the coupling coefficient r is so high that always the process is unstable; during the cutting process waviness develops not only on the surface of the work, but also on the surface of the grinding wheel. Therefore, grinding requires special study.

The main simplifying conditions of the described theory are: the variable part of the cutting force depends only on the y-component of the vibration, is proportional to it, the coefficient of proportionality r being a real number; and the vibratory system of the machine is linearized.

Concerning the methods of measurement of dynamics characteristics and of the calculation of the limit of stability, modifications have been proposed by Kudinov [15] and by Péters [14], which improve and simplify in some cases the described type of operations (*e.g.*, if the number of various directional orientations of the cutting process in the machine is small).

The authors of this paper are of the opinion that, under the present state of knowledge about self-excited vibration in machining from the point of view of the machine, it is sufficient to consider the vibratory parameters of the machine in relation to its design. We are able, on the basis of measured parameters of natural vibrations of a given machine, to indicate which of these parameters are mainly responsible for the actual stability of the machine and how the values of the individual parameters should be changed in order to raise the stability. We are not able to do that on the basis of drawings of the machine, as we cannot calculate the values of natural vibrations of the machine with sufficient accuracy. Neither are we able to predict, with sufficient certainty, the changes in the design of the measured machine so as to get the changes of natural vibration which we propose as the result of our calculation of the limit of stability. It would be useful, however, if we could indicate the dimensions and the type of design of the machine frame for a desired stability before we start with the design of it.

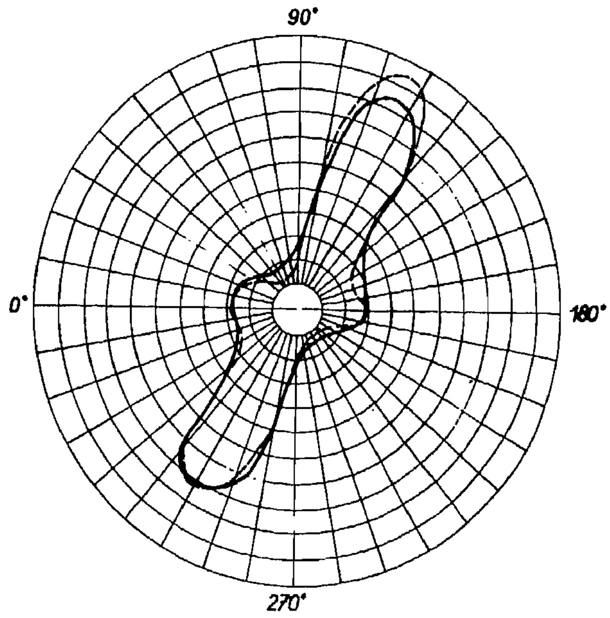

FIGURE 16

The authors do not think that the right approach to this problem would be the elaboration of methods of calculation for Ω_i, k_i, δ_i, α_i, which are necessary for the calculation of stability, directly from the drawings. Such calculation methods would be necessarily too tedious and would very probably never be sufficiently accurate. It seems more hopeful to look for a suitable combination of experimental work using simplified models of the parts of a future machine and performing machining tests with these models. Then by generalizing the results of the experiments, the future machine may be designed. The use of electronic computers for these calculations could be practical. Toward the elaboration of optimal practical methods of this kind future research work should be directed.

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