THE EFFECT ON THE CHATTER BEHAVIOUR OF MACHINE TOOLS OF CUTTERS WITH DIFFERENT HELIX ANGLES ON ADJACENT TEETH

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SUMMARY

The action of cutters having different helix angles on adjacent teeth is considered and a method of analysing their effect on machine tool chatter is presented. This analysis allows the full significance to be appreciated of the effect of this type of cutter on improving chatter performance. The improved performance obtainable is illustrated by results obtained with a slab milling cutter.

INTRODUCTION

The metal-removal rate on a machine tool is often limited by the onset of chatter, which causes poor surface finish, increased tool wear and greater power consumption; to avoid chatter a reduction in the rate of metal removal is usually necessary with a consequent loss of production. A great deal of research has therefore been directed to methods of increasing the chatter resistance of the machining process and various methods of overcoming chatter have been successful. These may be divided into two categories: those that increase the dynamic stiffness of the machine and those that affect the cutting forces by modification of the cutter. The latter have tended to involve essentially varying the pitch between successive teeth. Thus Slavicek(1) gives examples of the improvement obtained with alternating pitch cutters but also demonstrates the limitation of the improvement to a restricted speed range. An extension of alternating pitch cutters was then proposed by Opitz(2) whereby the pitch between successive teeth changed continuously, e.g. in a sinusoidal manner. However, only simple alternating pitch was considered in detail and the results again showed that the improvement was limited to a restricted speed range. To overcome this limitation Vanherk,(3) who also found the speed restrictions on alternating pitch, proposed cutters with varying helix angles which were considered as an infinite number of adjacent cutters with different amounts of pitch variation. Thus as a given pitch variation improves the performance over a particular speed range it was considered that having different amounts of pitch variation on the same cutter would allow a wider speed range to be covered. A groove cutter was described which gave considerably improved chatter-free performance. However, the true value of alternating helix angle cutters is not appreciated when considered in this way and their application in practice seems to have been limited to narrow groove cutters.

This paper is concerned with presenting, in descriptive form, a method of analysis of the alternating helix effect which shows that this is a powerful means of increasing the chatter-free performance of a wide range of multi-tooth cutting operations. A detailed theoretical treatment of the effect may be found in refs. 4 and 5. In particular, a slab mill is considered in detail and the improved machining performance obtained is described.

REGENERATIVE CHATTER

The theory of regenerative chatter has been well established by many authors(6, 7, 8) and may be summarized by reference to Fig. 1. Regenerative chatter arises when a small oscillation of a tooth results in a wave being left on the surface. The following tooth has to remove this wave and, depending on the width of cut, leaves a wave of smaller or greater amplitude. When the latter is the case each succeeding tooth leaves a wave of greater amplitude resulting in a vibration of large amplitude which is limited only by non-linearities in the machine response and the cutting process. The situation shown in Fig. 1 represents the conditions existing at the boundary of stability when each tooth leaves a wave of the same amplitude; any increase in the width of cut would result in the vibration amplitude increasing.

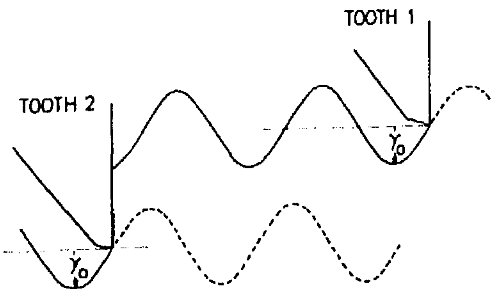

FIG. 1. Diagrammatic representation of the mechanism of regenerative chatter.

In this section the regenerative forces resulting from the wave left by a preceding tooth will be considered for cutters with both constant and non-constant helix angles. Initially conditions appropriate to the broaching operation as shown in Fig. 1 will be considered. Also for simplicity the cutting forces will be considered as being proportional to the cross-sectional area of the chip.

Constant Helix

For constant helix cutting the wave left by the preceding tooth will be parallel to the tooth in cut as may be seen in Fig. 2. The variation of the cross-sectional area of the chip

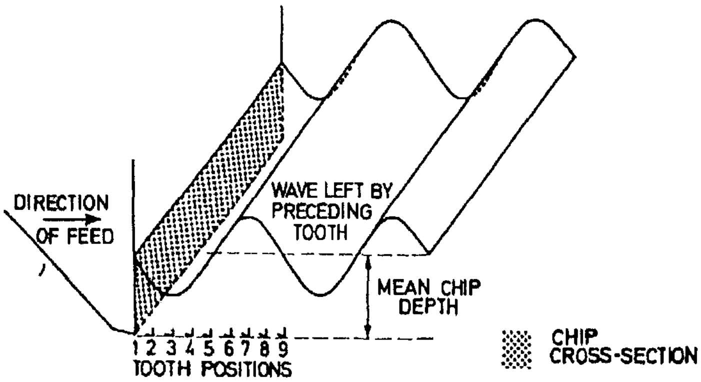

FIG. 2. Representation of constant helix cutting.

as the tooth removes a complete wave may be determined by considering the tooth in the positions numbered 1 to 9. In the position shown in full (i.e. position 1) the chip cross-section is shown shaded and is the mean chip cross-section. The chip cross-sections at the positions 1 to 9 are shown in Fig. 3 where the area over or under the mean is shown shaded. This is the important parameter since it is only oscillating forces that are involved and not the mean steady forces. The maximum variation of the chip area from the mean has been defined as 100 units and the variation of the area from the mean with tooth position is shown in Fig. 3.

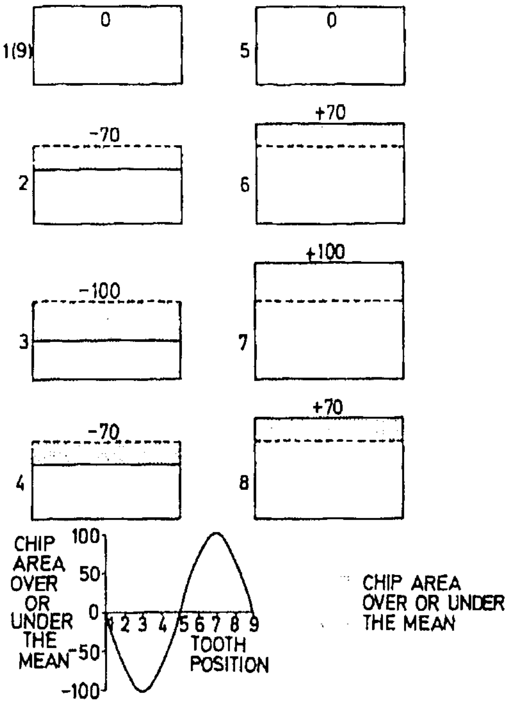

FIG. 3. Variation of chip cross-section for various tooth positions, constant helix cutting.

If the force is proportional to the cross-section area an oscillating force of amplitude proportional to 100 acts on the tooth.

Non-constant Helix

For comparison with the constant helix case the removal of a wave of the same amplitude will be considered and the units of area will be the same. The difference in helix angle results in the waves left by the preceding tooth being at an angle to the tooth in cut. Thus the tooth in cut will cross the waves left by the preceding tooth and may span more than one wave. As an example a difference in helix resulting in the tooth in cut crossing one and two-third waves is considered. This condition is shown in Fig. 4 with the chip cross-section shaded for the tooth position 1. The variation of the chip cross-section as the tooth moves through positions 1 to 9 may be determined and these chip cross-sections are shown in Fig. 5 with the area over or under the mean shown shaded. It may be seen that at each position considered the areas above and below the mean line are nearly equal and that the net area above or below the mean is greatly reduced compared with the constant helix case. The amplitude of the regenerative force has thus been greatly reduced as may be seen from Fig. 5. The amplitude of the oscillating area is now 16.5 compared with the 100 for the constant helix cutter.

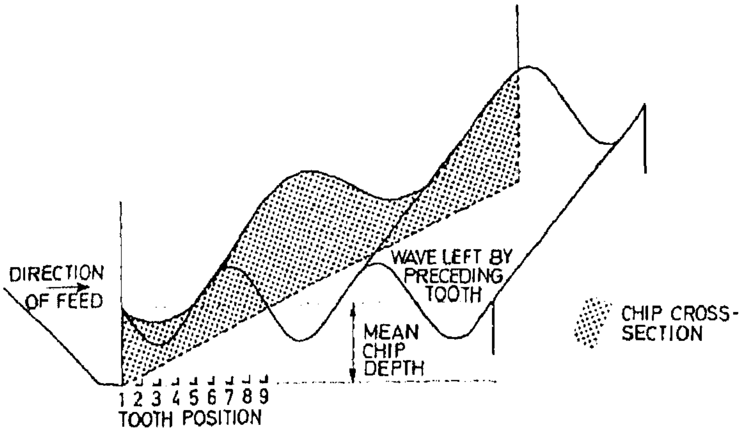

Fig. 4. Representation of non-constant helix cutting.

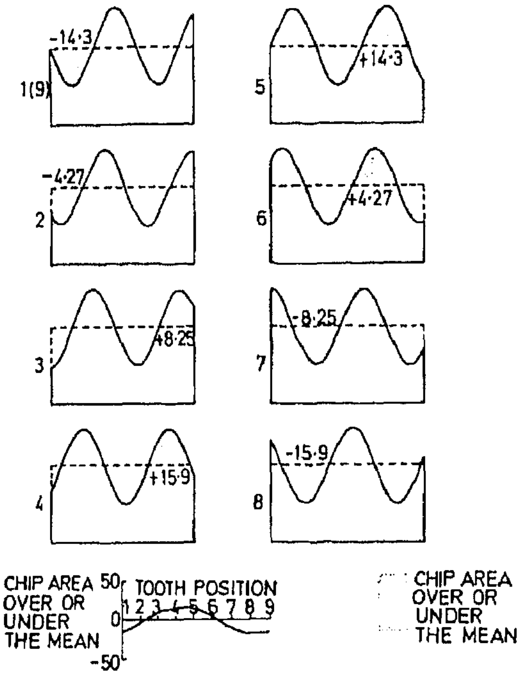

Fig. 5. Variation of chip cross-section for various tooth positions, non-constant helix cutting.

The example considered applies only to a particular helix difference resulting in one and two-thirds waves being crossed. However, for different wavelengths, which arise at different cutting speeds and vibration frequencies, and also for greater or smaller differences in helix angle, the number of waves crossed will vary. The reduction in the regenerative force amplitude also varies and may be shown(4, 5) to vary in the manner shown in Fig. 6. The amplitude of variation of chip area has again been taken as 100 for the constant helix case where no waves are crossed. The reduction of amplitude is then shown as a function of the number of waves crossed and when an exact number of waves is crossed the amplitude of regenerative force becomes zero. It should be noted that, provided at least one wave is crossed, a reduction in the regenerative force of a factor of approximately 5 is assured.

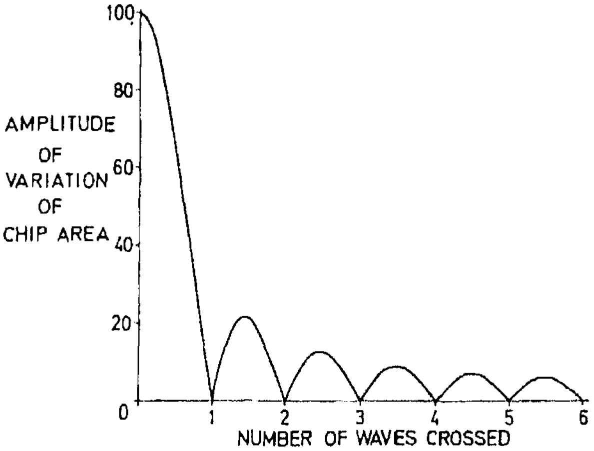

FIG. 6. Variation of regenerative force amplitude with number of wavelengths crossed.

STABILITY CHARTS

At this stage the analysis has been concerned only with the effect of non-constant helix angles on the regenerative force, whereas the effect on the chatter performance of a machine and cutter combination is the subject of interest. It is therefore necessary to summarize briefly the theory for the machine/constant helix case and for this purpose it will be assumed that the structure responds as a simple spring/mass damped system. The more complex situation of a multi-degree of freedom machine tool may be analysed in a similar manner.

The spring/mass damped system chosen is shown in Fig. 7 where, for convenience, the

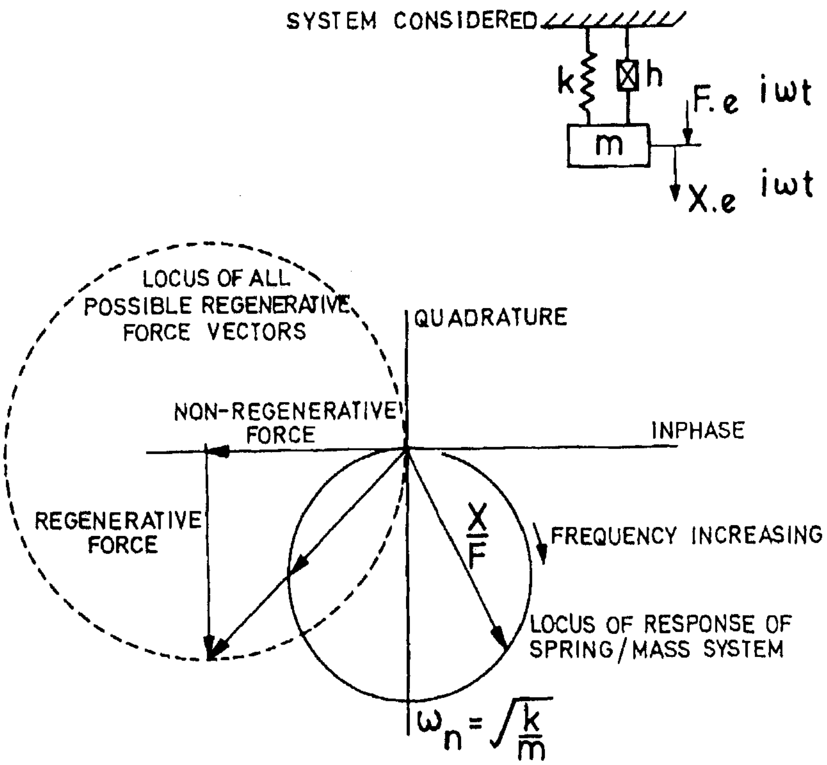

FIG. 7. Response of spring/mass system with vector diagram of oscillating cutting force.

damping is considered to be hysteretic. The response of such a system to an oscillating force is shown in Fig. 7 as a vector locus which in this case is part of a circle. The maximum amplitude of vibration for a force of constant amplitude would arise at the natural frequency $\omega_n = \sqrt{(k/m)}$ on the quadrature axis. The vector moves around the locus in a clockwise direction with increasing frequency of vibration.

At the stability boundary the oscillating forces may be determined by reference to Fig. 1. The forces on tooth 2 arise from the wave left by the previous tooth and also as a result of the tooth itself vibrating. (This latter force is not affected by varying the helix angle.) At the stability boundary both these forces are of the same amplitude for constant helix cutting and the force resulting from the tooth in cut is in antiphase to the vibration, i.e. is a vector along the negative real axis (Fig. 7). The regenerative force is of the same amplitude but may have any phase depending on the wavelength and tooth pitch. Thus its locus is a circle as shown in Fig. 7. The net oscillating force is the vector sum of these two forces.

For all the relevant conditions to be satisfied at a given cutting speed the response of the structure to the oscillating force must be such as to result in the amplitude already assumed for tooth 2. The equilibrium condition is found by considering increasing widths of cut, i.e. increasing chip cross-sections, until the oscillating forces are of such a magnitude as to give the required displacement. One further condition is required. The phase of the regenerative force at a given cutting speed and tooth pitch is governed by the vibration frequency. Thus the resultant force vector must intercept the response locus at this frequency. Because of these compatibility relations the width at which chatter occurs varies with the speed.

A non-dimensional stability chart for the spring/mass system considered is shown in Fig. 8. This diagram applies to both broaching and milling. The exact position of the lobes depends on the resonant frequency of the machine and on the tooth pitch. The absolute values of the widths of cut would depend on the dynamic stiffness of the machine in question and also on the chip area/cutting force relation.

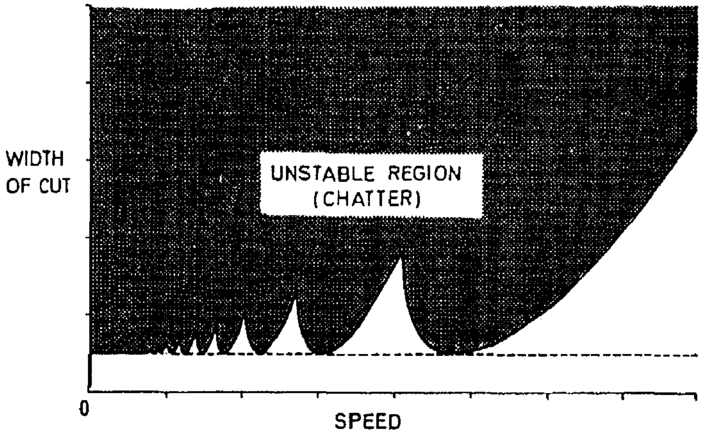

FIG. 8. Typical stability chart for constant helix cutting.

It should be noted that for a more complicated chip area/force relation the stability chart will be modified mainly in the lower-speed range.(8)

Non-constant Helix Broaching

When the same analysis is applied to non-constant helix cutting and the boundary of stability is considered, the tooth in cut has the same force antiphase to the motion, i.e. a vector along the negative real axis, but the amplitude of the regenerative force will, in general, have been reduced and thus the resultant force is also reduced. Furthermore, this tends to increase the frequency at which chatter must occur, resulting in a shorter wavelength and more waves being crossed so that the regenerative force may be reduced further. Thus to obtain chatter the resultant force must be increased and increasing the width of cut does not necessarily accomplish this. As the width of cut increases, the force resulting from the tooth in cut increases proportionately, but the regenerative force may be still further reduced.

The net result of all these effects is to eliminate chatter at lower speeds when the wavelengths are small and large numbers of waves are crossed. At the higher speeds, when chatter may arise, increasing the width of cut may eliminate chatter as the condition of a whole number of waves being crossed is reached.

A typical stability chart(4, 5) for the spring/mass system considered is shown in Fig. 9. The speed and width-of-cut axes are directly comparable with those in Fig. 8. Chatter is eliminated at the lower speeds and at speeds when chatter may occur an increase in the width of cut can be beneficial. The lowest speed at which chatter may occur is governed by the difference in helix angles, the resonant frequency of the machine, tooth pitch, machine stiffness and the chip area/cutting force relationship.

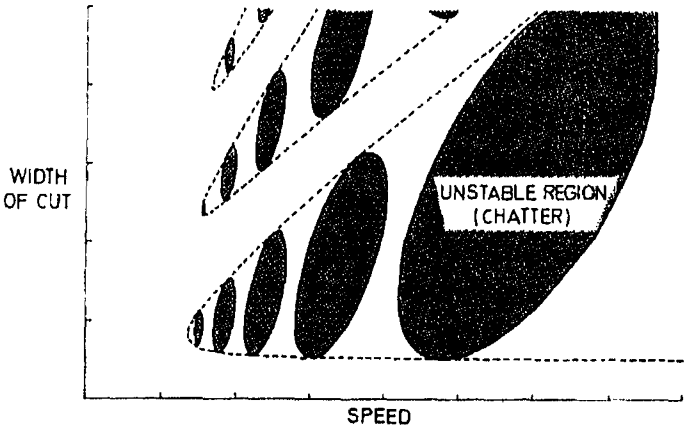

FIG. 9. Typical stability chart for non-constant helix broaching.

Non-constant Helix Milling

Milling differs from broaching in that for non-zero helix angles, large widths and small depths the effective width of cut is not the width of the workpiece. This may be demonstrated by reference to Fig. 10 which shows a conventional slab-milling operation. As a tooth moves round the arc of cut through positions 1–6 relative to the chip cross-section it effectively moves in a direction normal to the feed, i.e. across the arc of cut. The effective width of cut is thus the arc of cut and is constant depending on the depth of cut. The result of increasing the workpiece width is to increase the mean number of teeth in contact which increases the total chip cross-section in cut at any time. Thus for a standard milling cutter a stability chart as shown in Fig. 8 would be obtained.

For non-constant helix cutting the same reasoning applies as for broaching with the exception that an increase in the workpiece width does not reduce the regenerative force. The regenerative force is governed by the wavelength, the length of the arc of cut and the

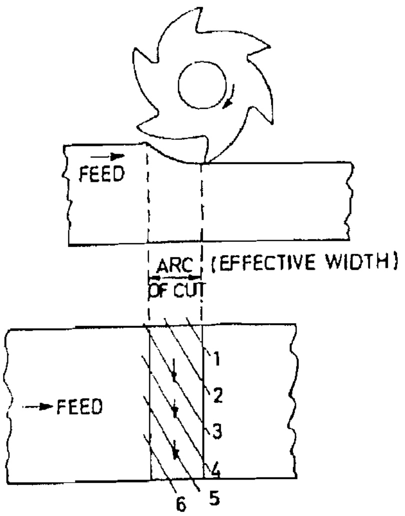

FIG. 10. Effective width of cut for slab milling.

difference in helix angle. An increase in the workpiece width produces an increase in the mean number of teeth in cut so that both the regenerative force and the force due to the tooth in cut are proportionately increased. Thus it is not now possible to increase the workpiece width and eliminate chatter. However, the number of waves crossed is now dependent on the wavelength and the length of the arc of cut. Thus, as the speed is reduced and the number of waves crossed increases, conditions arise, depending on the depth of cut, when the regenerative force is eliminated because an exact number of waves is crossed.

For a particular depth of cut a typical stability chart is shown in Fig. 11,(4, 5) whence it may be seen that at particular speeds chatter is eliminated and that, except for the higher

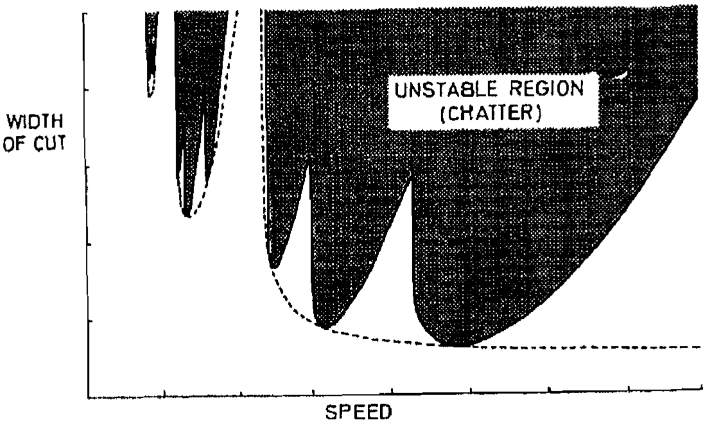

FIG. 11. Typical stability chart for non-constant helix milling.

speeds, an improvement of a factor of at least four will be obtained. This stability chart is directly comparable with that shown in Fig. 8. The positions of the dotted envelope curves are moved to the right for increased depth of cut and increased difference in helix angle.

Non-constant Speed Effect

There is another effect resulting fron non-constant helix milling which was ignored in the previous section. This may be explained with reference to Fig. 10. As the effective cutting direction is normal to the feed, the surface speed depends on both the rotational speed and the helix angle. Thus teeth with smaller helix angles are effectively moving at greater speeds across the arc of cut so that for non-constant helix cutters the surface speed of successive teeth is different. This results in the frequency of the regenerative force being different for successive teeth and will undoubtedly improve the chatter performance as both teeth will be excited away from the critical frequency.

The non-constant speed effect may be taken to its limit when the difference in cutting speeds is infinite. As the helix angle of a tooth tends to zero, the speed at which it moves across the arc of cut increases until for zero helix angle it moves infinitely fast. This has direct practical significance as may be seen from Fig. 12. Chatter marks are shown on the arc of cut which have been left by a tooth of zero helix angle and the tooth in cut has some positive helix angle. The chip cross-section is shown and as the tooth moves across the arc of cut the chip cross-section remains constant. Thus the regenerative effect is eliminated. (This does not strictly apply while the tooth is coming into full cut or whilst leaving.) However, there is a practical limitation to having a zero helix tooth as the effect of a tooth taking up a cut over the full workpiece width instantaneously produces a large force which would result in forced vibration. However, a compromise may be reached by having a suitably small helix angle.

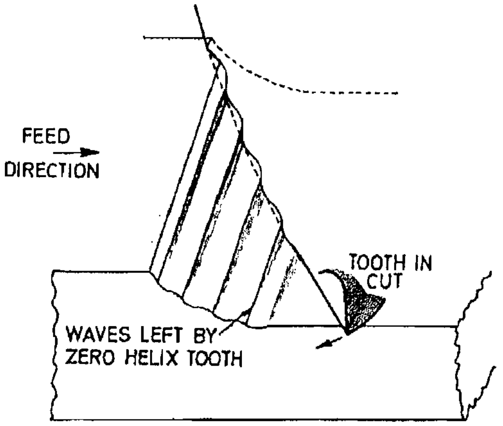

FIG. 12. Milling with non-constant helix when one helix is zero.

Tunable Alternating Pitch

For both broaching and milling there is one final effect which is beneficial. As was noted in the introduction, the effect of alternating pitch is very powerful at particular speeds and frequencies. Thus if for non-constant helix the workpiece width is symmetrically disposed about the constant pitch line, i.e. the mean pitch is constant, it is possible to impose alternating pitch by moving the position of the cutter laterally relative to the workpiece. This effect is, in fact, tunable because, as noted in the introduction, a particular pitch variation will be very effective at certain speed and frequency combinations. Thus if chatter did occur at a small width the cutting condition could be optimally tuned by lateral movement of the cutter until the pitch variation is the optimum for the existing speed and frequency.

For slab milling the situation is slightly more complicated since, as successive teeth are moving effectively at different speeds, the pitch is continuously changing. However, lateral movement of the cutter imposes different limits on the minimum and maximum pitch.

M.T.I.R.A. CUTTERS

The major effects noted are the subject of patents applied to all multi-tooth cutters and the non-constant cutting speed effect is the subject of a separate patent as this is not limited to multi-tooth cutting.

As an example of the application of the design parameters discussed, a slab mill will be considered having six teeth with alternating helix angles. This is cheaper and easier to manufacture than continuously varying helix, which is also unlikely to be more efficient since the difference in helix angles between successive teeth is the important parameter. The helix angles were chosen about a mean of 30° and four cutters were made with differences in helix angle of 1.5°, 3.0°, 5.0° and 10.0°. Thus the latter cutter had alternating helix angles of 25° and 35°. These cutters were compared with a constant 30° helix angle cutter with six teeth. All the cutters were manufactured with nominally the same rake and clearance angles and made from the same bar with identical heat treatment. The standard cutter and the 25°/35° cutter are shown in Fig. 13. The diameter and width of the cutters were both 4 in.

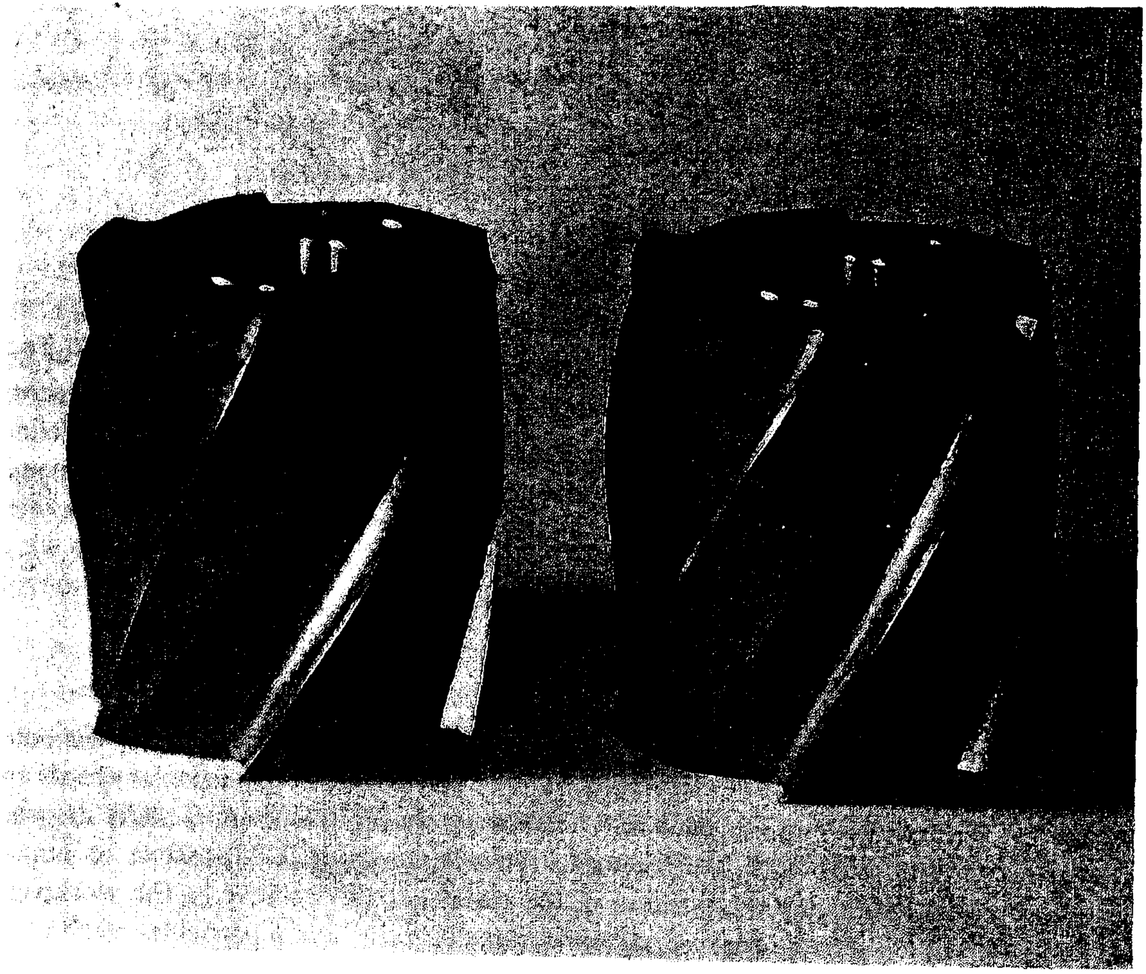

FIG. 13.

Cutting tests were conducted on taper workpieces to ascertain the widths at which chatter occurred for various speeds, feeds and depths of cut. The results obtained were all of the same general form as shown in Fig. 14 for a speed of 72 rev/min and a feed of $2\frac{3}{8}$

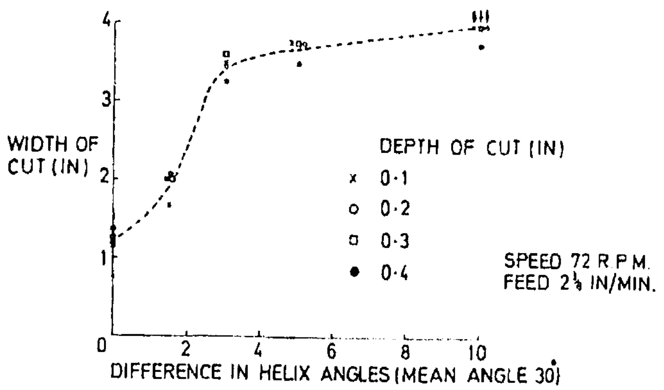

Fig. 14. Widths of cut at stability boundary for various differences in alternating helix angles.

in./min. The helix angle difference of 3° gave an improvement of a factor of three and the largest difference of 10° at the larger depths of cut gave factors greater than four. For some speeds the improvement with depth of cut was more pronounced.

For the smallest helix difference of 1½° the effect of lateral movement of the cutter on the chatter performance was investigated for a depth of cut of 0.2 in. The results are shown in Fig. 15 where it may be seen that the width of cut at chatter continuously increases until it is no longer possible to induce chatter within the limit of the cutter width. The effect may be expected to be more pronounced for greater differences in helix angle and for operations where the mean constant pitch position results in a low width of cut at chatter. This often is the case with flexible castings.

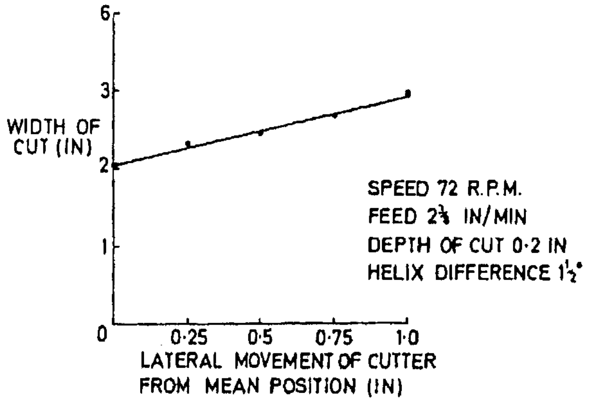

Fig. 15. Effect of lateral movement of cutter on the chatter performance.

Similar results have been obtained on several different machines and also with shell end mills when the equivalent to the width is the depth of cut. Cutters with one helix angle zero have been made and none have as yet chattered though, as was expected at larger widths of cut, a violent forced vibration arises. End mills have also been made with significant improvements in performance, particularly during slotting.

It is hoped that the cutters described will have a significant part to play in the elimination of chatter for many machining operations.

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